Deferred annuities with gender-neutral pricing: Benefitting most women without adversely affecting too many men

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Abstract

Many countries emphasize gender equality and ban gender-based annuity pricing, leading to more heterogeneous health characteristics and market inefficiency. Governments may respond by offering deferred annuities when annuitants' health characteristics are more similar at an earlier age. The two combined policies benefit most female annuitants without adversely affecting too many male annuitants, contrasting to the well-known result that banning gender-based pricing benefits all women but adversely affects all men, when only immediate annuities are available. Within each gender group, these two policy interventions benefit annuitants with average health, but adversely affect those on either end.

Keywords: heterogeneous health; asymmetric information; gender-neutral pricing; immediate annuity; deferred annuity

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1 Introduction

The quest for gender equality has been prevalent in many countries recently. In the area of economic policy, the European Union (EU) introduced new rules in 2012 to ban the use of gender-based pricing in insurance markets on the ground that "gender equality is regarded as a fundamental right."[1] Similarly, some states in the United States of America have banned the use of gender-based pricing in auto insurance and other insurance products.[2]

Another major global trend relevant to the current study is population aging. Facing rapid population aging, citizens in many countries need to have adequate financial resources to face the longer post-retirement days. Moreover, it is well known that the pay-as-you-go pension system adopted in some developed countries are not financially sustainable in the coming decades unless appropriate policy changes are implemented. Learning from the experience of these countries, many countries have chosen or are building up the defined-contribution retirement income system in the last few decades.[3] However, one disadvantage of this type of pension system is that the retirees have to bear the main responsibilities to insure against longevity risk (that is, the risk of outliving the available financial resources when they live longer than expected). Adopting appropriate retirement income protection policies will be beneficial to current and future retirees facing longer post-retirement years.

The annuity market, which provides an important function in helping retirees to insure against longevity risk, is affected by both trends mentioned above. In this paper, we focus on annuitization choices in countries adopting defined-contribution retirement income system and thus retirees have to deal with longevity risk. Information asymmetry between buyers and sellers is present in many annuity markets, and it usually leads to adverse selection, with higher-risk annuitants (who have longer life expectancy) buying more annuities. The buyers suffer from the resulting high price and low level of transactions (or even the collapse of the market in the extreme case). When appropriate, annuity providers use various observed characteristics (such as gender, age and race) of the buyers to reveal part of health-related

[1] Information about this decision by the EU can be found from the website: https://ec.europa.eu/commission/presscorner/detail/en/MEMO_12_1012.

[2] Many insurance policies in the US are regulated at the state, rather than federal, level. For example, the regulation by Montana can be found from: https://csimt.gov/laws-rules/advisory-memos/10272009_genderltc/. The list of US states that have banned gender-based premiums in auto insurance can be found from: https://www.insure.com/car-insurance/gender-auto-insurance-rates/.

[3] See Mandatory Provident Fund Schemes Authority (2015, Table 3.1) and OECD (2021, pp. 49-51 and Table 2.1).

information, leading to an outcome with less severe adverse selection (Hoy, 1982; Crocker and Snow, 1986). Before gender-neutral pricing is imposed, annuity providers in many countries price annuities differently for the two gender groups.

In this paper, we intend to study the economic impact of adopting gender-neutral pricing, as well as introducing deferred annuities, on the equilibrium annuity prices and the annuitants' welfare. We interpret that the first policy is imposed because of social and political factors, and economic policy-makers usually take this policy as given. On the other hand, economic policy-makers may decide whether offering deferred annuities or not. These two policies, which apparently look quite different in their main objectives, can be linked through the lens of the heterogeneity of buyers' health characteristics and the resulting severity of adverse selection. Banning gender-based pricing in the annuity market combines male and female annuitants in the same pool. The heterogeneity of health characteristics of the new pool generally increases because the life expectancies of men and women are different. While it is well known that banning gender-based pricing leads to a higher degree of severity of adverse selection, the main contributions of this paper arise mainly from analyzing the additional policy of introducing deferred annuities.[4] Working-age deferred annuities are offered to the annuitants at an earlier age (say, around age 50) when their health characteristics are more similar. As a result, the degree of heterogeneity of annuitants' health characteristics is reduced by this policy. Given the above social and economic trends in many countries, it is important to analyze the combined effects of these two policy changes, each of which has different effects on the annuity prices and annuitants' welfare.

We adopt a two-gender version of the three-period model of Brugiavini (1993) to address these policy issues. The model allows us to obtain sharp analytical results regarding the equilibrium annuity prices. However, it is difficult to obtain analytical results regarding annuitants' welfare because of the mutual dependence of deferred and immediate annuity markets, and we have to use numerical analysis for these issues. Using theoretical and numerical approaches, we analyze (a) the annuitants' choices and the equilibrium annuity prices, and (b) the annuitants' welfare. We find that the annuitization choices are systematically affected by the gender gaps in health and

[4] The deferred annuity is generally "purchased today but does not pay until the annuitant survives to a pre-specified age" (Chen et al., 2020, p. 373). There are two major types of deferred annuities, namely the working-age deferred annuity and advanced-age deferred annuity (Milevsky, 2005; Brown, 2008; Chen et al., 2020). We focus on the working-age deferred annuity in this paper, because the less heterogeneity of survival probabilities of the annuitants in their prime working ages (say, between 45 to 50), relative to that at retirement, is a key element of our policy analysis.

wealth, as well as the two policies regarding whether gender-based pricing is banned and whether deferred annuities are introduced. In particular, we show that advantageous selection may arise in the deferred annuity market. We also compare the equilibrium deferred annuity price with the immediate annuity price for each gender group under gender-based pricing and before deferred annuities are introduced. Based on these results, we study two interesting dimensions of annuitants' welfare: comparison between the two gender groups, and comparison among annuitants with different health levels within each gender group. We find systematic differences in annuitants' welfare in both dimensions.

The remaining sections are organized as follows. Section 2 summarizes the related literature. Section 3 sets up a three-period model with gender gaps in health and wealth. Section 4 analyzes the effect of banning gender-based pricing in an economy with only immediate annuities. Section 5 examines the effect of introducing deferred annuities under gender-neutral pricing on the annuity prices. Section 6 examines the effects of these policies on annuitants' welfare, based on computational analysis. Section 7 concludes.

2 Related literature

This paper is related to the literature about the effects of banning gender-based pricing in insurance markets with information asymmetry. Asymmetric information appears in many insurance markets, which usually results in adverse selection (Rothschild and Stiglitz, 1976). In the presence of adverse selection, insurance providers could use risk-related observed characteristics, such as age, gender and race, as imperfect signals to partially reveal the information of risk types. Hoy (1982) and Crocker and Snow (1986) show that using imperfectly categorizing risks based on observed characteristics may enhance market efficiency and lead to welfare improvement. In particular, gender-based pricing is adopted by many annuity providers to alleviate the inefficiency caused by asymmetric information of life expectancies (or mortality rates), because women have a higher life expectancy than men. However, in recent decades, social and political consideration on gender equality in many countries have led to the prohibition on the use of gender in insurance markets. Finkelstein et al. (2009) and Aquilina et al. (2017) provide the evidence that banning gender-based pricing leads to market efficiency loss and redistributive effects of benefitting female annuitants but adversely affecting male annuitants.

Besides adopting characteristics-based prices, providing deferred annuities is another way to alleviate the annuity market inefficiency caused by asymmetric information (Brugiavini, 1993; Sheshinski, 2007; Brown, 2008). Deferred annuities are provided at an earlier period when buyers are more similar and have less private information about their future health conditions. Therefore, the heterogeneity of health among the pool of deferred annuity buyers is reduced. Direr (2010) and Sheshinski (2010) further provide welfare analysis in a more complicated environment in which longevity risk and another risk (such as future income or health expenditure risk) coexist. Motivated by the observed global trends, this paper examines the combined effects of providing deferred annuities and banning gender-based prices. We extend the model of Brugiavini (1993) to an economy with two gender groups and obtain new results complementary to the above papers.

3 The model

In this paper we consider the policies of introducing deferred annuities and imposing gender-neutral pricing. Brugiavini (1993) uses a three-period model to examine issues related to deferred annuities,[5] but she does not consider gender differences. We retain the main elements of her model but modify it by incorporating gender differences in health and wealth.

The sequence of events in the model is described as follows. The annuitants live in Periods 0 and 1 with certainty, but some of them may not survive to Period 2. In Period 0, the government provides deferred annuities that can be purchased by the members of a defined-contribution pension scheme using their accumulated contributions.[6] Insurance companies provide immediate annuities in Period 1, which is the early stage of retirement (corresponding to the period from ages 65 to 85). Period 2 is the later stage of retirement (corresponding to the period from age 85 onwards). Annuitants survive to Period 2 with some probability, denoted by θ, where $\theta \in [\theta_L, \theta_H]$

[5]To focus on the main issue of adverse selection, the model in Brugiavini (1993) does not address annuitants' liquidity concern. In this paper, we analyze the effects of introducing deferred annuities and banning gender-based pricing when the annuitants use their defined-contribution pension account balance to purchase deferred annuities. In this policy environment, since annuitants cannot use the lock-in retirement wealth for other purchases until the retirement age, it is not misleading in using the simplifying assumption of ignoring liquidity constraints.

[6]In line with the policy focus of this paper, we assume that deferred annuities are provided by the government or a statutory body. Lau and Zhang (2023, Section 2.1) discuss and summarize observed public annuity policies in various economies. While our analytical results hold whether deferred annuities are provided by the government or the private sector, we examine only public deferred annuities because the provision of these annuities is more consistent with the policy focus of this paper.

with $0 \leq \theta_L < \theta_H \leq 1$. Both the deferred annuity and immediate annuity provide survival-contingent payouts in Period 2.

The features described above are similar to those in Brugiavini (1993). We now incorporate gender differences in health and wealth. For convenience, we assume that there are equal number of male and female annuitants.[7]

3.1 Heterogeneous health levels with a gender gap

In our model, health level is denoted by θ, the probability of surviving to Period 2. We assume that the annuitants of each gender group are heterogeneous in θ. In Period 0, information about θ has not been revealed to the annuitants. In Period 1, θ is revealed to individual annuitants but is unknown to the annuity providers. The revelation process of θ follows Brugiavini (1993); in particular, annuitants of each gender group are assumed to be identical in health conditions at Period 0.

We incorporate gender differences in health in our model. The probability density function of θ of gender i is denoted by $h(\theta|i)$, where $i = f$ refers to female annuitants and $i = m$ refers to male annuitants. To capture the gender gap in health, we assume $h(\theta\,|f)$ monotone likelihood-ratio (MLR) dominates $h(\theta\,|m)$:

$$\frac{h(x\,|f)}{h(x\,|m)} > \frac{h(y\,|f)}{h(y\,|m)} \tag{1}$$

for $x > y$, where x and y are two arbitrary levels of $\theta \in [\theta_L, \theta_H]$.[8] The MLR assumption is a frequently-used assumption in the literature on information asymmetry (Milgrom, 1981).

Panel A of Figure 1 presents the probability density functions $h(\theta\,|f)$ and $h(\theta\,|m)$. Panel B shows that $\frac{h(\theta|f)}{h(\theta|m)}$ is an increasing function of θ satisfying the MLR property (1).

[Insert Figure 1 here.]

Although θ is private information, gender can be observed by annuity providers and reveals partial information of θ. Condition (1) can be written as $\frac{h(x|f)}{h(y|f)} > \frac{h(x|m)}{h(y|m)}$, which means that a female annuitant is more likely to be

[7]Lau et al. (2023) consider a model with possibly unequal sizes of male and female annuitants, and show that some of the equations relevant for this paper can easily be extended to a model with unequal sizes of the two gender groups. Since the relative size of these two groups is not crucial for the issues investigated in this paper, we keep the model simple by assuming equal group size.

[8]An alternative way to express condition (1) is $\frac{d}{d\theta}\left[\frac{h(\theta|f)}{h(\theta|m)}\right] > 0$.

healthier than a male annuitant. In particular, condition (1) leads to female annuitants having a higher average survival probability than male annuitants:

$$\overline{\theta}^f \equiv \int_{\theta_L}^{\theta_H} \theta h(\theta\,|f)\,d\theta > \int_{\theta_L}^{\theta_H} \theta h(\theta\,|m)d\theta \equiv \overline{\theta}^m. \tag{2}$$

The proof, which is presented in the Online Appendix, is well known. United Nations (2019, Table 2) shows the remaining life expectancy at age 65 is 18.3 years for women, higher than that for men (15.6 years). The empirical evidence is consistent with (2).

The annuitants and annuity providers know the average survival probability of each gender in Period 0. In particular, the annuity providers find it profitable to use this information to price their products, if this practice is allowed.

3.2 Homogeneous wealth levels with a gender gap

There is also a gender gap in wealth, besides that in health. Moreover, retirement wealth levels for each gender group are heterogeneous, and health and wealth are correlated. While the most empirically relevant specification is that health and wealth of each gender group are heterogeneous and these two variables are correlated, the analysis based on this general specification is quite complicated and it is less likely to obtain sharp results. Since asymmetric information of heterogeneous health levels is the main source of adverse selection in the annuity market (Einav and Finkelstein, 2011), we assume health heterogeneity within a gender group and the gender gap in health, as in Section 3.1 above. On the other hand, we simplify the model by assuming away the heterogeneity in wealth within a gender group while only keeping the wealth gender gap.

Under this specification, annuitants of the same gender have the same level of retirement wealth, but there is a gender gap in wealth:

$$w^m = gw^f > w^f, \tag{3}$$

where w^i is the retirement wealth level of a gender-i annuitant's defined-contribution pension account, and $g > 1$ to capture the wealth gap between gender groups. The gender gap in retirement wealth is observed in the survey of Health and Retirement Study (HRS). According to Wave 13 of the HRS, male pensioners contribute 6,850 US Dollars to their individual accounts annually on average, which is more than 1.4 times female pensioners' annual contributions (4,742 US Dollars).[9] Moreover, assumption (3) is consistent

[9] We eliminate the outliers of this variable by excluding the contributions below the bottom 1% and above the top 1%.

with the gender gap in wage income, which can be interpreted as a proxy for retirement wealth. Blau and Kahn (2017) survey the literature on gender gap in income; according to the evidence in 2014, full-time female workers earned about 79 percent of that of the male workers.

3.3 Annuitants' optimal choices

In Period 0, the government provides deferred annuities and charges $\frac{p_\delta}{(1+r)^2}$ for each unit of the deferred annuity, where r is the risk-free interest rate. Gender-i annuitants use their retirement wealth to purchase δ^i units of deferred annuities. In Period 1, the levels of θ is revealed to the annuitants. In the same period, the insurance companies provide the immediate annuity. The price of one unit of immediate annuity is $\frac{p_\alpha}{1+r}$.[10] After knowing his or her level of θ, each annuitant purchases α^i_θ units of immediate annuities. The deferred and immediate annuity contracts are of the non-exclusive type with linear pricing, following the specification used in Abel (1986), Brugiavini (1993) and Hosseini (2015).

We use backward induction to solve the annuitants' optimization problems. In Period 2, a surviving annuitant of gender i receives annuity payouts and consumes. In Period 1, given any level of deferred annuity choice δ^i, a gender-i annuitant with survival probability θ chooses immediate annuity α^i_θ to maximize

$$U^i_\theta \equiv U\left(c^i_{1\theta}, c^i_{2\theta}; \theta\right) = u\left(c^i_{1\theta}\right) + \frac{\theta}{1+\rho} u\left(c^i_{2\theta}\right), \tag{4}$$

subject to the budget constraints:

$$c^i_{1\theta} = (1+r)\left[w^i - \frac{p_\delta}{(1+r)^2}\delta^i\right] - \frac{p_\alpha}{1+r}\alpha^i_\theta, \tag{5}$$

and

$$c^i_{2\theta} = \delta^i + \alpha^i_\theta, \tag{6}$$

where $\delta^i \geq 0$, $\alpha^i_\theta \geq 0$, ρ is the subjective discount rate, and $c^i_{j\theta}$ is the annuitant's level of consumption expenditure in Period j ($j = 1, 2$). We assume that the utility function $U\left(c^i_{1\theta}, c^i_{2\theta}; \theta\right)$ in (4) is homothetic, with the property that the marginal rate of substitution is a homogeneous function of degree 0:

$$\frac{\partial U\left(tc^i_{1\theta}, tc^i_{2\theta}; \theta\right)/\partial c^i_{1\theta}}{\partial U\left(tc^i_{1\theta}, tc^i_{2\theta}; \theta\right)/\partial c^i_{2\theta}} = \frac{\partial U\left(c^i_{1\theta}, c^i_{2\theta}; \theta\right)/\partial c^i_{1\theta}}{\partial U\left(c^i_{1\theta}, c^i_{2\theta}; \theta\right)/\partial c^i_{2\theta}} \tag{7}$$

[10] Note that p_δ and p_α are defined as above such that the (normalized) prices of deferred and immediate annuities can be directly compared even though the two financial products are offered at different time periods.

for $t > 0$.[11] We also make the standard assumptions that $u(c)$ is strictly concave and $\lim_{c\to 0} u^{'}(c) = \infty$ hold.

After substituting (5) and (6) into (4), it is straightforward to obtain

$$\frac{\partial U_\theta^i}{\partial \alpha_\theta^i} = \frac{\theta}{1+\rho} u'\left(\delta^i + \alpha_\theta^i\right) - \frac{p_\alpha}{1+r} u'\left((1+r)\,w^i - \frac{p_\delta}{1+r}\delta^i - \frac{p_\alpha}{1+r}\alpha_\theta^i\right). \quad (8)$$

Based on (8), it can be shown that if $\theta > \theta_\alpha^i$, where

$$\theta_\alpha^i = \frac{p_\alpha (1+\rho)\, u'\left((1+r)\,w^i - \frac{p_\delta}{1+r}\delta^i\right)}{(1+r)\, u'\left(\delta^i\right)}, \quad (9)$$

then $\alpha_\theta^{i*} > 0$ is an interior solution which is determined according to[12]

$$\frac{p_\alpha}{1+r} u'\left((1+r)\,w^i - \frac{p_\delta}{1+r}\delta^i - \frac{p_\alpha}{1+r}\alpha_\theta^{i*}\right) = \frac{\theta}{1+\rho} u'\left(\delta^i + \alpha_\theta^{i*}\right). \quad (10)$$

Differentiating (10) with respect to θ leads to

$$\frac{d\alpha_\theta^{i*}}{d\theta} = \frac{-\frac{1}{1+\rho} u'\left(\delta^i + \alpha_\theta^{i*}\right)}{(\frac{p_\alpha}{1+r})^2 u''\left[(1+r)\,w^i - \frac{p_\delta}{1+r}\delta^i - \frac{p_\alpha}{1+r}\alpha_\theta^{i*}\right] + \frac{\theta}{1+\rho} u''\left(\delta^i + \alpha_\theta^{i*}\right)} > 0. \quad (11)$$

On the other hand, if $\theta \le \theta_\alpha^i$, then $\alpha_\theta^{i*} = 0$. Note that we use * to represent individual optimal choices or the equilibrium prices.

In Period 0, anticipating that α_θ^{i*} will be chosen in Period 1, a gender-i annuitant chooses δ^i to maximize the expected value of U_θ^i,

$$\int_{\theta_L}^{\theta_H} U_\theta^i h(\theta\,|i)\, d\theta. \quad (12)$$

The optimal choice, δ^{i*} is characterized by

$$\int_{\theta_L}^{\theta_H} \frac{p_\delta}{1+r} u'\left((1+r)\,w^i - \frac{p_\delta}{1+r}\delta^{i*} - \frac{p_\alpha}{1+r}\alpha_\theta^{i*}\right) h(\theta\,|i) d\theta$$

[11] The property of homotheticity is satisfied when the utility function is additively separable over time, as in (4), with CRRA specification for $u(c)$ in (32). This specification has been commonly used in the literature (such as Abel, 1986; Brown, 2001; Hosseini, 2015).

[12] In the simpler environment in Section 4 in which only immediate annuities are available, we express the dependence of the optimal choice of immediate annuity purchase on the annuity price ($\widehat{p}^i$ or $\widehat{p}$) and the annuitant's wealth (w^i) explicitly. When deferred annuities are also available, it can be seen from (10) that α_θ^{i*} depends on p_α, p_δ, δ^i and w^i. To avoid lengthy expression we simply write α_θ^{i*} and assume that the implicit dependence of α_θ^{i*} on other variables is understood.

$$= \int_{\theta_L}^{\theta_H} \frac{\theta}{1+\rho} u'\left(\delta^{i*} + \alpha_\theta^{i*}\right) h(\theta\,|i) d\theta, \tag{13}$$

where α_θ^{i*} is determined according to (10) if $\theta > \theta_\alpha^i$, or $\alpha_\theta^{i*} = 0$ otherwise. As in Brugiavini (1993), we simplify the optimization problem by assuming that consumption choice in Period 0 is irrelevant in the annuitants's objective function.

3.4 The zero-profit conditions

We assume both the annuity insurance companies (providing immediate annuities) and the government (providing deferred annuities) follow the zero-profit condition that the present discounted value of total annuity payment equals to the total premium received. In the literature (such as Abel, 1986; Villeneuve, 2003; Hosseini, 2015), it is usually assumed that there are free entry into and exit from the private annuity market, and the zero-profit condition is reasonable in this environment. Regarding the deferred annuity, we assume that the government does not have the profit motive in providing this product. Instead, they take a financially neutral position of making zero profit. As an example, Sweden has a public annuity program called Premium Pension. The annual financial statements of the Premium Pension scheme show that its net income is more or less equal to zero every year.[13]

In the immediate annuity market, the insurance companies receive immediate annuity premium $\int_{\theta_L}^{\theta_H} \frac{p_\alpha}{1+r} \alpha_\theta^f h(\theta\,|f)\, d\theta + \int_{\theta_L}^{\theta_H} \frac{p_\alpha}{1+r} \alpha_\theta^m h(\theta\,|m) d\theta$ from all annuitants in Period 1. The expected value of survival-contingent payments that the companies distribute to all surviving annuitants in Period 2 is $\int_{\theta_L}^{\theta_H} \theta \alpha_\theta^f h(\theta\,|f)\, d\theta + \int_{\theta_L}^{\theta_H} \theta \alpha_\theta^m h(\theta\,|m) d\theta$. Under the zero-profit condition, the equilibrium value of the immediate annuity price (p_α^*) and the optimal choices (α_θ^{f*} and α_θ^{m*}) are related according to

$$p_\alpha^* = \frac{\int_{\theta_L}^{\theta_H} \theta \alpha_\theta^{f*} h(\theta\,|f)\, d\theta + \int_{\theta_L}^{\theta_H} \theta \alpha_\theta^{m*} h(\theta\,|m) d\theta}{\int_{\theta_L}^{\theta_H} \alpha_\theta^{f*} h(\theta\,|f)\, d\theta + \int_{\theta_L}^{\theta_H} \alpha_\theta^{m*} h(\theta\,|m) d\theta}. \tag{14}$$

On the other hand, the government receives deferred annuity premium $\frac{p_\delta}{(1+r)^2}\delta^f + \frac{p_\delta}{(1+r)^2}\delta^m$ from all annuitants in Period 0, since all annuitants of the same gender purchase the same amount of deferred annuities. In Period 2, the government distributes survival-contingent payments $\int_{\theta_L}^{\theta_H} \theta \delta^f h(\theta\,|f)\, d\theta + \int_{\theta_L}^{\theta_H} \theta \delta^m h(\theta\,|m) d\theta$ to all surviving annuitants. Under the zero-profit condi-

[13] See, for example, Swedish Pension Agency (2020, p. 5).

tion, the equilibrium value of the deferred annuity price (p_δ^*) is given by

$$p_\delta^* = \frac{\int_{\theta_L}^{\theta_H} \theta \delta^{f*} h(\theta | f) d\theta + \int_{\theta_L}^{\theta_H} \theta \delta^{m*} h(\theta | m) d\theta}{\int_{\theta_L}^{\theta_H} \delta^{f*} h(\theta | f) d\theta + \int_{\theta_L}^{\theta_H} \delta^{m*} h(\theta | m) d\theta}$$

$$= \frac{\delta^{f*} \int_{\theta_L}^{\theta_H} \theta h(\theta | f) d\theta + \delta^{m*} \int_{\theta_L}^{\theta_H} \theta h(\theta | m) d\theta}{\delta^{f*} + \delta^{m*}} = \frac{\delta^{f*}\overline{\theta}^f + \delta^{m*}\overline{\theta}^m}{\delta^{f*} + \delta^{m*}}, \quad (15)$$

after using (2).

4 Imposing gender-neutral pricing when only immediate annuities are available

Gender-based pricing in the annuity market is currently adopted in several countries, such as Australia and Singapore. On the other hand, more and more countries (such as the EU countries after 2012) have already banned the use of gender in pricing insurance products. The effect of imposing gender-neutral pricing in the annuity market is a major issue that this paper focuses on.

Before examining the outcome under the policy intervention of introducing deferred annuities together with imposing gender-neutral pricing, we first consider the environment without deferred annuities in this section. In Section 4.1, we consider this economy when annuity providers offer separate financial products to annuitants of the two gender groups and price the products differently. In Section 4.2, we consider the outcome when the government bans gender-based pricing in annuities.

In the following analysis, we will use $\widehat{\cdot}$ to represent the corresponding variables before the deferred annuity is introduced ($\delta^i = 0$ for both gender groups in the model).

4.1 The reference economy before gender-based pricing is banned

We express an annuitant's purchase of immediate annuities under gender-based pricing, $\widehat{\alpha}_\theta^i(\widehat{p}^i, w^i)$, as a function of annuity price ($\widehat{p}^i$) and annuitant's wealth (w^i). It is easy to see from (10) that the optimal annuitization amount $\widehat{\alpha}_\theta^{i*}(\widehat{p}^i, w^i)$ is chosen according to the first-order condition

$\frac{\widehat{p}^i}{1+r}u'\left((1+r)\,w^i - \frac{\widehat{p}^i}{1+r}\widehat{\alpha}_\theta^{i*}\left(\widehat{p}^i, w^i\right)\right) = \frac{\theta}{1+\rho}u'\left(\widehat{\alpha}_\theta^{i*}\left(\widehat{p}^i, w^i\right)\right)$.[14] Similar to (14), the equilibrium immediate annuity price of gender i ($\widehat{p}^{i*}$) is given by

$$\widehat{p}^{i*} = \frac{\int_{\theta_L}^{\theta_H} \theta\widehat{\alpha}_\theta^{i*}\left(\widehat{p}^{i*}, w^i\right) h(\theta\,|i)d\theta}{\int_{\theta_L}^{\theta_H} \widehat{\alpha}_\theta^{i*}\left(\widehat{p}^{i*}, w^i\right) h(\theta\,|i)d\theta}. \tag{16}$$

Based on the formula $cov\left(\theta, \widehat{\alpha}_\theta^{i*}\left(\widehat{p}^{i*}, w^i\right)\right) = \int_{\theta_L}^{\theta_H} \theta\widehat{\alpha}_\theta^{i*}\left(\widehat{p}^{i*}, w^i\right) h(\theta|i)d\theta - \overline{\theta}^i E\left(\widehat{\alpha}_\theta^{i*}\left(\widehat{p}^{i*}, w^i\right)\right)$, (16) can also be written as

$$\widehat{p}^{i*} - \overline{\theta}^i = \frac{cov\left(\theta, \widehat{\alpha}_\theta^{i*}\left(\widehat{p}^{i*}, w^i\right)\right)}{E(\widehat{\alpha}_\theta^{i*}\left(\widehat{p}^{i*}, w^i\right))}. \tag{17}$$

Equation (17) has a useful interpretation when there is adverse selection in the annuity market. The source of adverse selection is that higher-risk annuitants buying more annuities in the presence of asymmetric information, according to (11). Therefore, $\widehat{\alpha}_\theta^{i*}\left(\widehat{p}^{i*}, w^i\right)$ and θ are positively correlated, leading to a positive term on the right-hand side (RHS) of (17), which can be interpreted as a measure of the severity of adverse selection.[15] Equation (17) shows that in the presence of adverse selection, the annuity price is higher than the actuarially fair price (which is equal to the average survival probability of the annuitants).

4.2 Imposing gender-neutral pricing

When gender-neutral pricing is imposed, the optimal choice of an annuitant's immediate annuity purchase, $\widehat{\alpha}_\theta^{i*}\left(\widehat{p}, w^i\right)$, is determined according to

$$\frac{\widehat{p}}{1+r}u'\left((1+r)\,w^i - \frac{\widehat{p}}{1+r}\widehat{\alpha}_\theta^{i*}\left(\widehat{p}, w^i\right)\right) = \frac{\theta}{1+\rho}u'\left(\widehat{\alpha}_\theta^{i*}\left(\widehat{p}, w^i\right)\right). \tag{18}$$

As a result, the equilibrium price of the immediate annuity with gender-neutral pricing ($\widehat{p}^*$) is defined as

$$\widehat{p}^* = \frac{\int_{\theta_L}^{\theta_H} \theta\widehat{\alpha}_\theta^{f*}\left(\widehat{p}^*, w^f\right) h(\theta\,|f)d\theta + \int_{\theta_L}^{\theta_H} \theta\widehat{\alpha}_\theta^{m*}\left(\widehat{p}^*, w^m\right) h(\theta\,|m)d\theta}{\int_{\theta_L}^{\theta_H} \widehat{\alpha}_\theta^{f*}\left(\widehat{p}^*, w^f\right) h(\theta\,|f)d\theta + \int_{\theta_L}^{\theta_H} \widehat{\alpha}_\theta^{m*}\left(\widehat{p}^*, w^m\right) h(\theta\,|m)d\theta}. \tag{19}$$

[14] This first-order condition, in an economy with only immediate annuities and with gender-based pricing, can be regarded as a special case of (10), where $\delta^i = 0$ and p_α is replaced by $\widehat{p}^i$.

[15] Similar interpretation has appeared in Villeneuve (2003) and Lau and Zhang (2023).

It is useful to define

$$\widehat{\beta}^f = \frac{E(\widehat{\alpha}_\theta^{f*}(\widehat{p}^*, w^f))}{E(\widehat{\alpha}_\theta^{f*}(\widehat{p}^*, w^f)) + E(\widehat{\alpha}_\theta^{m*}(\widehat{p}^*, w^m))} \tag{20}$$

as the share of purchased immediate annuities by female annuitants, where

$$E\left(\widehat{\alpha}_\theta^{i*}\left(\widehat{p}^*, w^i\right)\right) = \int_{\theta_L}^{\theta_H} \widehat{\alpha}_\theta^{i*}\left(\widehat{p}^*, w^i\right) h(\theta\,|i) d\theta. \tag{21}$$

The effect of banning gender-based pricing is given in the following proposition. The proof is presented in Appendix A.

Proposition 1. *Consider a three-period model with the assumptions of gender gap in health (1), gender gap in wealth (3) and homothetic utility function (7). When only immediate annuities are available, banning gender-based pricing leads to*

(a) an increase in the equilibrium annuity price for male annuitants and a decrease for female annuitants

$$\widehat{p}^{m*} < \widehat{p}^* < \widehat{p}^{f*}, \tag{22}$$

where $\widehat{p}^{m*}$ *and* $\widehat{p}^{f*}$ *are given in (16),* $\widehat{p}^*$ *is given in (19); and*

(b)

$$\widehat{p}^* - \frac{1}{2}\left(\overline{\theta}^f + \overline{\theta}^m\right) = \widehat{\lambda}^{wg} + \widehat{\lambda}^{bg}, \tag{23}$$

where

$$\widehat{\lambda}^{wg} = \widehat{\beta}^f \frac{cov(\theta, \widehat{\alpha}_\theta^{f*}(\widehat{p}^*, w^f))}{E(\widehat{\alpha}_\theta^{f*}(\widehat{p}^*, w^f))} + (1 - \widehat{\beta}^f) \frac{cov(\theta, \widehat{\alpha}_\theta^{m*}(\widehat{p}^*, w^m))}{E(\widehat{\alpha}_\theta^{m*}(\widehat{p}^*, w^m))} > 0 \tag{24}$$

and

$$\widehat{\lambda}^{bg} = \frac{\left(\overline{\theta}^f - \overline{\theta}^m\right)\left[E(\widehat{\alpha}_\theta^{f*}(\widehat{p}^*, w^f)) - E(\widehat{\alpha}_\theta^{m*}(\widehat{p}^*, w^m))\right]}{2\left[E(\widehat{\alpha}_\theta^{f*}(\widehat{p}^*, w^f)) + E(\widehat{\alpha}_\theta^{m*}(\widehat{p}^*, w^m))\right]}. \tag{25}$$

According to part (a) of Proposition 1, the equilibrium annuity price under gender-neutral pricing ($\widehat{p}^*$) is lower than $\widehat{p}^{f*}$ but higher than $\widehat{p}^{m*}$.[16] Thus, banning gender-based pricing in the annuity market benefits the female annuitants (who are the high-risk group) but adversely affects the low-risk

[16] Even though Proposition 1(a) is quite intuitive, this result is generally difficult to prove, because of the mutual dependence of annuity price and annuitization choices (in, for example, $\widehat{p}^*$, $\widehat{\alpha}_\theta^{f*}$ and $\widehat{\alpha}_\theta^{m*}$ for the annuity market under gender-neutral pricing). The use of MLR assumption (1) and homotheticity property (7) are crucial in our proof.

male group. This result is quite intuitive and consistent with the empirical evidence (Finkelstein et al., 2009; Aquilina et al., 2017). Some analysis in the subsequent sections will make use of this result.

Part (b) of Proposition 1 compares the equilibrium annuity price under gender-neutral pricing and the actuarially fair price. According to (23), the difference between these two prices can be decomposed into two terms: the within-group term ($\widehat{\lambda}^{wg}$) and the between-group term ($\widehat{\lambda}^{bg}$). In particular, comparing (24) with (17), we see that $\widehat{\lambda}^{wg}$ can be traced to the severity of adverse selection of each of the two gender groups. We will refer to the decomposition of the difference of equilibrium annuity price and the actuarially fair price in subsequent analysis when we compare with the outcome after deferred annuities are introduced.

5 Introducing deferred annuities after gender-neutral pricing is imposed

After 2012, the insurance companies in EU countries are required to adopt gender-neutral pricing. Countries planning to join the EU are also required to follow. Moreover, it is likely that some other countries will follow the lead of EU and ban gender-based pricing in annuity markets in the future. The analysis in the previous section has shown the impact of this policy on the equilibrium immediate annuity prices.

Taking the adoption of gender-neutral pricing as given, we analyze in this section the effect of introducing another policy intervention: offering deferred annuities. A main reason of considering this policy is to see whether it could offset the effect of banning gender-based pricing on the heterogeneity of annuitants' health characteristics. Pooling annuitants of the two gender groups together, when gender-neutral pricing is imposed, leads to *more heterogeneity* in the pool of annuitants. If adverse selection is an important factor in the annuity market, then more heterogeneity generally leads to higher severity of averse selection and efficiency loss. Following this line of thought, we are interested to examine whether adopting policies reducing heterogeneity among the pool of annuitants may lead to a counter-balancing effect. A potential policy option is to offer the annuitants the deferred annuity at an earlier age when their health characteristics are *less heterogeneous*.

We now consider the economy after gender-neutral pricing is imposed. Before deferred annuities are introduced, the equilibrium price of immediate annuities in Period 1 under gender-neutral pricing is $\widehat{p}^*$. Based on $\widehat{p}^*$ in (19), the government introduces deferred annuities in Period 0 at a lower price such that

$$p_\delta < \widehat{p}^*, \tag{26}$$

and the zero-profit condition (15) holds. In the above model, both deferred and immediate annuities perform similar functions in insuring against longevity risk, but the immediate annuities are offered at a later time after private health information is revealed. If (26) does not hold and the price of deferred annuities is equal to or even higher than the immediate annuity price, then annuitants will have no incentive to purchase annuities offered earlier.[17] In order to have an effective public deferred annuity policy, the deferred annuity offered by the government has to satisfy (26).

5.1 The equilibrium annuity prices

The following proposition analyzes the annuitants' choices and market outcome after introducing deferred annuities under gender-neutral pricing. The proof is presented in Appendix B.

Proposition 2. *Consider a three-period model with the assumptions of gender gap in health (1), gender gap in wealth (3) and homothetic utility function (7). After the government introduces deferred annuities that satisfy (26), there exists an equilibrium such that*

(a) transactions in deferred and immediate annuities coexist;

(b)

$$p_\delta^* < p_\alpha^*, \tag{27}$$

where p_δ^* and p_α^* are determined according to (15) and (14); and

(c)

$$p_\delta^* - \frac{1}{2}\left(\overline{\theta}^f + \overline{\theta}^m\right) = \frac{\left(\overline{\theta}^f - \overline{\theta}^m\right)\left(\delta^{f*} - \delta^{m*}\right)}{2\left(\delta^{f*} + \delta^{m*}\right)}. \tag{28}$$

Part (a) of Proposition 2 presents the coexistence of deferred and immediate annuities under gender-neutral pricing. We show in Appendix B that male annuitants with the highest value of survival probability θ always have the incentive to buy the immediate annuity (by comparing the marginal benefit of buying the first unit of immediate annuity with the marginal cost). Thus, introducing the deferred annuity does not completely crowd out the immediate annuity market. This result contrasts with the outcome of complete crowding out of the immediate annuity market under gender-based pricing (Brugiavini, 1993). Similarly, we show that under the supposition of

[17] In other words, deferred annuity purchases and the zero-profit condition are consistent only with $p_\delta < \widehat{p}^*$ in (26), but inconsistent with $p_\delta \geq \widehat{p}^*$.

no purchase of the deferred annuity, the annuitants will always buy immediate annuities to insure against longevity risk in all possible states of θ in Period 1. Thus, integrating over all these health states, the marginal benefit of buying the first unit of deferred annuity is always higher than the marginal cost, contradicting the supposition of no transaction in the deferred annuity.

Part (b) shows that the equilibrium price of deferred annuities is lower than that of immediate annuities. As shown in Appendix B, if $p^*_\delta \geq p^*_\alpha$, then annuitants prefer to wait to make the annuity purchase decisions in Period 1, because they can receive updated health information and purchase at a lower (or equal) cost. Thus, the coexistence of deferred and immediate annuity transactions is not consistent with $p^*_\delta \geq p^*_\alpha$. Given that the annuitants have already purchased deferred annuities in Period 0 based on the average survival probability by gender, only annuitants with high survival probability θ have the residual demand for immediate annuities in Period 1. As a result, there is more distortion in the immediate annuity market, which leads to a higher immediate annuity price at the equilibrium. This result is useful in understanding the welfare effect of introducing deferred annuities, which will be presented in Section 6.

Part (c) shows that the difference between the equilibrium price of deferred annuity and the actuarially fair price is determined by the interaction of two factors: gender gaps in health and in purchased levels. As implied by assumption (1), female annuitants have a higher average survival probability than male annuitants. At the group level, female annuitants are classified as the high-risk group and male annuitants as the low-risk group. On the other hand, the purchased levels of deferred annuities are determined endogenously. The gender gap in annuity purchase $(\delta^{f*} - \delta^{m*})$ is affected by health and wealth factors. Compared with female annuitants, higher wealth level of male annuitants leads them to buy more annuities, but their relatively low survival probabilities lead them to buy less. According to (28), the equilibrium price of the deferred annuity could be lower than the actuarially fair price when male annuitants (the low-risk group) buy more deferred annuities, leading to advantageous selection, instead of adverse selection.

There is an interesting comparison of (28) and (23) of this paper, as well as (17) of Lau et al. (2023), in which the decomposition formula is first stated. In Table 1, $\lambda = p^* - \frac{1}{2}\left(\overline{\theta}^f + \overline{\theta}^m\right)$ is used as a measure of the severity of adverse selection in each of these three cases, as well the decomposition of this term into the within-group and between-group components $(\lambda = \lambda^{wg} + \lambda^{bg})$.

[Insert Table 1 here.]

For the voluntary purchase of immediate annuities corresponding to (23) before the deferred annuity is introduced, or the mandatory purchase of annuities under the partial waiver policy corresponding to (17) of Lau et al. (2023), the within-group term is positive, indicating that annuitants with higher θ buy more annuities.[18] Adverse selection is present. On the other hand, when annuitants choose deferred annuities in Period 0, the information of survival probability in Period 1 has not yet been revealed. Thus, all annuitants within each gender group are homogeneous in health levels and purchase the same level of deferred annuities. As a result, the within-group term in (28) is zero. The between-group term for the three cases has the similarity that it depends on the product of gender gaps in health and in annuity purchased levels. Given the assumptions of gender gaps in health and wealth, it is possible that men (the low-risk group) buy more annuities than women, leading to a negative between-group term. In the mandatory annuity plan considered in Lau et al. (2023), women purchase less annuities because of the partial waiver clause for low-income annuitants. Hence, the between-group term is always negative. On the other hand, the sign of the between-group term for the two annuity markets considered in this paper is ambiguous.

Combining these two components, we conclude that the severity of adverse selection is less likely to be negative for voluntary purchase of immediate annuities before the deferred annuity is introduced. On the other hand, the severity of adverse selection is more likely to be negative in the other two cases, leading to advantageous selection.[19]

5.2 Comparing with the reference economy

In Section 5.1, we have analyzed the effect of introducing deferred annuities when gender-based pricing is banned. We now compare the outcome under these two policy interventions with the reference economy that only immediate annuities under gender-based pricing are available. The results of how the equilibrium prices change due to combining these two policies are summarized in the next proposition, and the proof is given in Appendix C.

Proposition 3. *Compared with the equilibrium prices of the immediate annuity markets under gender-based pricing when deferred annuities are not available, introducing deferred annuities under gender-neutral pricing leads to*

[18] As shown in Lau et al. (2023), the within-group term is positive for the mandated annuity purchase case when health and wealth are positively correlated, but is zero when they are uncorrelated.

[19] The results presented in Section 6 are consistent with these predictions.

(a)

$$p_\delta^* < \widehat{p}^{f*}; \tag{29}$$

and (b)

$$p_\delta^* < \widehat{p}^{m*}, \tag{30}$$

if the sufficient condition

$$\overline{\theta}^f - \overline{\theta}^m < \frac{cov\left(\theta, \widehat{\alpha}_\theta^{m*}\left(\widehat{p}^{m*}, w^m\right)\right)}{E(\widehat{\alpha}_\theta^{m*}\left(\widehat{p}^{m*}, w^m\right))} \tag{31}$$

holds.

The intuition of part (a) of Proposition 3 is relatively straightforward. Because of adverse selection, we obtain $\widehat{p}^{f*} > \overline{\theta}^f$. When annuitants of the two gender groups are pooled under gender-neutral pricing, p_δ^* is determined according to (15). Given that $\overline{\theta}^f > \overline{\theta}^m$ and p_δ^* lies between $\overline{\theta}^m$ and $\overline{\theta}^f$, we obtain $p_\delta^* \leq \overline{\theta}^f$. Combining these two results, it is easy to conclude that when compared with the annuity price for female annuitants under gender-based pricing ($\widehat{p}^{f*}$) initially, the equilibrium deferred annuity price p_δ^* is always lower.

The impact on male annuitants is more complicated, as p_δ^* is higher, but not lower, than $\overline{\theta}^m$. One of the two favorable factors relevant to female annuitants is not relevant for male annuitants. As a result, (30) does not always hold. We show that (31) is a sufficient condition for (30), based on the comparison of $\widehat{p}^{m*} - \overline{\theta}^m$ and $p_\delta^* - \overline{\theta}^m$. According to (17), the difference between $\widehat{p}^{m*}$ and $\overline{\theta}^m$ is $\frac{cov(\theta, \widehat{\alpha}_\theta^{m*}(\widehat{p}^{m*}, w^m))}{E(\widehat{\alpha}_\theta^{m*}(\widehat{p}^{m*}, w^m))}$, which measures the severity of adverse selection in the immediate annuity market for men under gender-based pricing. More severe adverse selection in this market means that $\widehat{p}^{m*}$ is much higher than $\overline{\theta}^m$. Based on (15), $p_\delta^* - \overline{\theta}^m$ is smaller than $\overline{\theta}^f - \overline{\theta}^m$, the gender gap in health. A small gender gap in health implies that p_δ^* is not much higher that $\overline{\theta}^m$. According to part (b) of Proposition 3, if the severity of adverse selection in the immediate annuity market for men under gender-based pricing is higher than the gender gap in health, then (30) holds.

Proposition 3 suggests that it is also possible for male annuitants to benefit from the introduction of deferred annuities under gender-neutral pricing, in the sense that p_δ^* is lower than $\widehat{p}^{m*}$ of the reference economy. This result contrasts with Proposition 1, which states that imposing gender-neutral pricing leads to a higher price ($\widehat{p}^* > \widehat{p}^{m*}$) and thus adversely affects all male annuitants. Proposition 3 is useful when we examine the effects of these policy changes on the welfare of annuitants of the two gender groups.

6 Effects of policy changes on the annuitants

In Sections 4 and 5, we have examined the impact of imposing gender-neutral pricing and introducing deferred annuities on the equilibrium annuity prices. We now conduct the complementary analysis regarding the effects of these two policies on the welfare of annuitants.

As shown in the previous sections, the annuitization choices and annuity prices of both markets are mutually dependent, according to (10), (13), (14) and (15). The mutual dependence among these variables in both markets makes it difficult to obtain analytical results on annuitants' welfare. Given this difficultly, we adopt a computational approach to analyze these issues. Section 6.1 presents the elements of the computational analysis. Section 6.2 conducts the relatively simple analysis of only imposing gender-neutral pricing (corresponding to Section 4). Section 6.3 considers the effects on annuitants' welfare of both policy interventions of introducing the deferred annuity and imposing gender-neutral pricing (corresponding to Section 5). Section 6.4 examines the effects on annuitants in countries with different gender gaps in health and wealth.

6.1 Elements of computational analysis

We adopt the commonly-used CRRA specification for the within-period utility function (as in Abel, 1986; Hosseini, 2015),

$$u(c) = \begin{cases} \frac{c^{1-\phi}-1}{1-\phi}, & \phi \neq 1 \\ ln(c), & \phi = 1 \end{cases}, \tag{32}$$

where ϕ (> 0) is the coefficient of relative risk aversion and the homotheticity property (7) is satisfied. In the baseline case, we choose $r = 0.3$, $\rho = 0.28$, $\phi = 0.5$, $w^f = 100$ and $g = 1.44$.

To specify the gender gap in health, we adopt the survey data of self-reported probability of living to age 85 from Wave 1 of the RAND Health and Retirement Study (HRS). The question is "how about the chances that you will live to be 85 or more?" and the choices (in percentage) are 0, 10, 20, ... and 100.[20] The mean and standard deviation of self-reported survival probability by gender are given in Table 2. We fit the self-reported survival probabilities of the two gender groups as normal distributions. To approximate the survey results in Table 2 in which the provided choices for the

[20]In more recent waves, the survey question was changed to "how about the chances that you will live to another ten years." We use the question in Wave 1, which is more relevant to obtain the information about θ that denotes the survival probability from the early retirement stage (65-85) to the late retirement stage (85+).

question are discrete values with increments of 10 percent, we assume that $\overline{\theta}^f = 0.5$ and $\overline{\theta}^m = 0.4$ in the baseline case of our computational analysis. Also, the standard deviation values for annuitants of the two gender groups are assumed to be equal at $\sigma^f = \sigma^m = \sigma = 0.3$.

[Insert Table 2 here.]

For the interval of θ, we use $\theta^L = 0.001$ and $\theta^H = 0.999$.[21] In order to keep the total probabilities equal to 1 within the restricted interval, we use the truncated normal distribution with probability density function for gender i, which is defined by

$$h(\theta\,|i) = \frac{exp\left[-\frac{1}{2}\left(\frac{\theta-\mu^i}{\sigma^i}\right)^2\right]}{\int_{\theta_L}^{\theta_H} exp\left[-\frac{1}{2}\left(\frac{\theta-\mu^i}{\sigma^i}\right)^2\right] d\theta}, \tag{33}$$

where $\sigma = 0.3$, $\mu^f = 0.5$ and $\mu^m = 0.338$ are used to ensure $\overline{\theta}^f = 0.5$ and $\overline{\theta}^m = 0.4$ for the truncated distributions. The fitted survival probability distributions are presented in Panel A of Figure 1. As shown in Panel B of Figure 1, these two probability density functions satisfy the MLR property (1).

On the other hand, the gender gap in wealth is assumed to be $g = 1.44$, based on Wave 13 of HRS.

Using the above specification, we obtain the gender-based actuarially fair prices are $\overline{\theta}^f = 0.5$ for female annuitants and $\overline{\theta}^m = 0.4$ for male annuitants, and gender-neutral actuarially fair price is $\frac{1}{2}(\overline{\theta}^f + \overline{\theta}^m) = 0.45$ for the whole population. Before any policy intervention, the equilibrium prices of immediate annuity are $\widehat{p}^{f*} = 0.6493$ for female annuitants and $\widehat{p}^{m*} = 0.5776$ for male annuitants, indicating the presence of adverse selection.

6.2 Effects of banning gender-based pricing

We first focus on the policy intervention of banning gender-based pricing. Under this policy, the value of the equilibrium annuity price is $\widehat{p}^* = 0.6123$, which is lower than $\widehat{p}^{f*} = 0.6493$ but higher than $\widehat{p}^{m*} = 0.5776$. This is consistent with part (a) of Proposition 1. In terms of part (b) of Proposition 1, annuitization levels within each gender group increase with survival

[21] The outcomes of non-existence or unboundedness of some equilibrium values may appear in computation when $\theta^L = 0$. We use the interval of $[0.001, 0.999]$ instead of $[0, 1]$ to avoid this inconvenience.

probabilities, leading to the within-group term $\widehat{\lambda}^{wg} = 0.16 > 0$. On the other hand, male annuitants with lower average survival probability purchase more annuities than female annuitants with $E(\widehat{\alpha}_{\theta}^{m*}(\widehat{p}^{*}, w^{m})) = 73.83 > 68.95 = E(\widehat{\alpha}_{\theta}^{f*}(\widehat{p}^{*}, w^{f}))$, leading to the between-group term $\widehat{\lambda}^{bg} = -0.0017 < 0$. As a result, $\widehat{\lambda}^{wg} + \widehat{\lambda}^{bg}$ in (23) is still positive in the baseline case, implying that the equilibrium immediate annuity price is higher than the actuarially fair price under gender-neutral pricing.

To examine the effect of banning gender-based pricing on the annuitant's welfare, we define the utility difference before and after this policy change as

$$\widehat{U}_{\theta}^{i*}\left(w^{i}, \widehat{p}^{*}\right) - \widehat{U}_{\theta}^{i*}\left(w^{i}, \widehat{p}^{i*}\right), \tag{34}$$

where $\widehat{U}_{\theta}^{i*}(w^{i}, \widehat{p}^{i*})$ is the annuitant's maximized utility level in Period 1 under gender-based pricing, and $\widehat{U}_{\theta}^{i*}(w^{i}, \widehat{p}^{*})$ is the corresponding level under gender-neutral pricing, both before deferred annuities are available.

We also apply the concept of equivalent wealth to examine the effect of the new policy on annuitants' welfare.[22] Define $ew_{\theta}^{i}\left(gn\&nda\right)$ as the hypothetical wealth level of a gender-i annuitant with θ such that[23]

$$\widehat{U}_{\theta}^{i*}\left(ew_{\theta}^{i}\left(gn\&nda\right), \widehat{p}^{i*}\right) = \widehat{U}_{\theta}^{i*}\left(w^{i}, \widehat{p}^{*}\right), \tag{35}$$

where the left-hand side (LHS) term is the maximized utility level with wealth $ew_{\theta}^{i}\left(gn\&nda\right)$ in the reference economy (with gender-based pricing and no deferred annuity). Suppose that instead of imposing gender-neutral pricing, the government adopts the policy of directly transferring cash to (or withdrawing cash from) the annuitants.[24] Equation (35) has the interpretation that the effect of imposing gender-neutral pricing on the welfare of an annuitant of gender i with θ is equivalent to that of cash transfer/withdrawal at the level of $ew_{\theta}^{i}\left(gn\&nda\right) - w^{i}$ if there is no policy change. When $ew_{\theta}^{i}\left(gn\&nda\right) - w^{i} > 0$, the government has to transfer wealth to annuitants to make them indifferent

[22] Various versions of the utility-based measure of equivalent wealth have been used in Mitchell et al. (1999) and Brown (2001). As an example, Brown (2001) uses the annuity equivalent wealth in his analysis.

[23] The definition of equivalent wealth involves both initial and final states, as indicated in (35). In this paper, we use equivalent wealth in two different comparisons, both involving "gender-based pricing and no deferred annuity" as the initial state. To avoid using lengthy expressions, we only specify the final state in $ew_{\theta}^{i}\left(gn\&nda\right)$ in (35) and $ew_{\theta}^{i}\left(gn\&da\right)$ in (38), assuming that the initial state is understood.

[24] In practice, it is difficult for the government to transfer the correct amount of cash to (or to withdraw the correct amount from) the annuitants, especially in the presence of asymmetric information about θ. The hypothetical policy of cash transfer/withdrawal is used as a metaphor to help understand the definition of equivalent wealth.

to the policy change. This implies that imposing gender-neutral pricing benefits annuitants of gender i with θ. On the other hand, $ew_{\theta}^{i}\left(gn\&nda\right)-w^{i}<0$ implies that imposing gender-neutral pricing adversely affects these annuitants.

For each gender group, we plot the utility difference (34) against θ in Panel A of Figure 2, and $ew_{\theta}^{i}\left(gn\&nda\right)-w^{i}$ (i.e., the difference of equivalent wealth and actual wealth of a gender-i annuitant with θ) against θ in Panel B. The red solid line and the blue dashed line represent welfare changes for female and male annuitants, respectively. It can be seen from Panel A or Panel B that imposing gender-neutral pricing benefits all female annuitants, while adversely affects all male annuitants.

[Insert Figure 2 here.]

One advantage of using equivalent wealth, rather than utility difference, is that we can develop the corresponding measure for the whole group.[25] We define aggregate equivalent wealth for gender-i annuitants as

$$EW^{i}\left(gn\&nda\right)=\int_{\theta_L}^{\theta_H} ew_{\theta}^{i}\left(gn\&nda\right)h(\theta|i)d\theta, \tag{36}$$

which is the integral of $ew_{\theta}^{i}\left(gn\&nda\right)$. We obtain $\frac{EW^{f}(gn\&nda)-w^{f}}{w^{f}}=1.45\%$ and $\frac{EW^{m}(gn\&nda)-w^{m}}{w^{m}}=-1.10\%$, indicating that the above policy benefits the female group but adversely affects the male group at the aggregate level.[26] These different welfare effects of the two gender groups can be traced to part (a) of Proposition 1.

6.3 Effects of both policy interventions

We now study the effects of both policy interventions of introducing deferred annuity and imposing gender-neutral pricing.

When deferred annuities are introduced after banning gender-based pricing, we find that in the baseline case, all annuitants buy deferred annuities in Period 0, but only healthier retirees have the residual demand for immediate annuity in Period 1. Specifically, only male annuitants with $\theta\geq 0.818$ purchase immediate annuities in the baseline case.[27] Because of the gender

[25] Note that utility level is ordinal but equivalent wealth is cardinal.

[26] Given that w^{f} and w^{m} are different, it is more appropriate to use percentage difference, instead of absolute difference, in presenting the magnitude of the policy effect.

[27] As shown in Table 4 below, some female annuitants also purchase immediate annuities when the gender gap in health is large.

gap in health, female annuitants find the gender-neutral deferred annuity price (p_δ^*) more attractive and are more likely to purchase adequate amount of deferred annuities in Period 0 and thus are less likely to have residual demand for immediate annuities in Period 1. Therefore, introducing deferred annuities further distorts the immediate annuity market, leading to a higher immediate annuity price ($p_\alpha^* = 0.9252 > 0.4494 = p_\delta^*$), which is consistent with part (b) of Proposition 2. Regarding part (c) of Proposition 2, male annuitants purchase more deferred annuities than female annuities ($\delta^{m*} = 117.93 > 115.16 = \delta^{f*}$) in the baseline case. Thus, the equilibrium deferred annuity price is lower than the actuarially fair price in the baseline case, according to (28).

When compared with the annuity prices under gender-based pricing, the sufficient condition of (31) in Proposition 3 holds with $\overline{\theta}^f - \overline{\theta}^m = 0.1 < 0.18 = \frac{cov(\theta,\widehat{\alpha}_\theta^{m*}(\widehat{p}^{m*},w^m))}{E(\widehat{\alpha}_\theta^{m*}(\widehat{p}^{m*},w^m))}$, which leads to (30). Hence, annuitants of both gender groups face a cheaper deferred annuity price, when compared with the reference economy.

Similar as above, we define another utility difference

$$U_\theta^{i*}\left(w^i, p_\delta^*, p_\alpha^*\right) - \widehat{U}_\theta^{i*}\left(w^i, \widehat{p}^{i*}\right), \tag{37}$$

where $U_\theta^{i*}\left(w^i, p_\delta^*, p_\alpha^*\right)$ is the annuitant's maximized utility level under gender-neutral pricing, when both deferred and immediate annuities are available. We also define $ew_\theta^i\left(gn\&da\right)$ as the hypothetical level of wealth of gender i annuitants with θ such that

$$\widehat{U}_\theta^{i*}\left(ew_\theta^i\left(gn\&da\right), \widehat{p}^{i*}\right) = U_\theta^{i*}(w^i, p_\delta^*, p_\alpha^*), \tag{38}$$

which means that the annuitant's utility level with wealth $ew_\theta^i\left(gn\&da\right)$ under the reference economy equals that with wealth w^i under both policy interventions. The amount of cash transfer at the level of $ew_\theta^i\left(gn\&da\right) - w^i$ can be used to measure the effect of both policy interventions.

The utility difference in (37) and $ew_\theta^i\left(gn\&da\right) - w^i$ in (38) are plotted against θ in Panels C and D of Figure 2. Unlike Panel A or B, Panel C or D shows that welfare changes are more similar for annuitants of the two gender groups. Poor-health female annuitants lose but good-health female annuitants benefit. In terms of male annuitants, they can be divided into three groups: male annuitants from average-health group benefit, poor-health and good-health male annuitants are adversely affected by the policy.

After deferred annuities are introduced under gender-neutral pricing, annuitants' behavior and welfare are summarized as follows. Annuitants purchase deferred annuities in Period 0 based on the survival probability distribution of their gender group. In Period 1, poor-health annuitants of each gender group whose survival probabilities are much lower than the mean will be adversely affected due to the inability to sell the over-purchased deferred annuities. We label this effect as the *over-purchase effect*. For good-health male annuitants, although their residual demand can be satisfied by purchasing immediate annuities in Period 1, they are still adversely affected by the high equilibrium immediate annuity price. The effect is called the *intensified distortion effect*. Note that the policy does not adversely affect good-health female annuitants in the baseline case, because no female annuitant buys the immediate annuity. After knowing their values of θ, annuitants with average health level neither find the purchased amount of deferred annuities very high nor need to purchase a lot of immediate annuities. As a result, they are not substantially affected by the over-purchase or intensified distortion effects, and they benefit because the effect due to a lower equilibrium deferred annuity price ($p_\delta^* = 0.4494 < 0.6493 = \widehat{p}^{f*}$ and $p_\delta^* = 0.4494 < 0.5776 = \widehat{p}^{m*}$) dominates.

Similar with (36), the aggregate effect of the combined policy interventions is measured by

$$EW^i\left(gn\&da\right) = \int_{\theta_L}^{\theta_H} ew_\theta^i\left(gn\&da\right) h(\theta|i)d\theta. \tag{39}$$

We obtain $\frac{EW^f(gn\&da)-w^f}{w^f} = 5.46\%$ and $\frac{EW^m(gn\&da)-w^m}{w^m} = 0.13\%$. Since the average-health annuitants of either gender are more likely to get benefit,[28] and the proportion of annuitants with average health is usually large (under the assumption of normal distribution), the aggregate welfare effect for either gender group is positive.[29] Moreover, the female group benefits more substantially than the male group.

6.4 Roles of gender gaps in health and wealth

In light of different gender gaps in health and wealth in different countries, we now examine the effects of introducing deferred annuities with gender-neutral pricing under different values of $\overline{\theta}^f - \overline{\theta}^m$ and g.

First, we examine how the gender gap in wealth affects equilibrium prices and the welfare of annuitants. We adopt $g = 1$, 1.2, 1.44 and 2, while the val-

[28]The sufficient condition (31) is satisfied in the benchmark case. As a result, the deferred annuity price under gender-neutral pricing is cheaper than $\widehat{p}^{m*}$ according to part (b) of Proposition 3, leading to the beneficial welfare effect for average-health male annuitants.

[29]Note that the aggregate welfare effect for male annuitants may be negative in other situations, as shown in Tables 3 and 4.

ues of other parameters remain unchanged. As shown in Table 3, when g increases (say, male annuitants' wealth increases and female annuitants' wealth is unchanged), annuitants of both gender groups purchase larger amounts of the deferred annuity. Moreover, the difference $\delta^{f*} - \delta^{m*}$ decreases (and becomes negative when $g = 1.44$ or $g = 2$), exerting downward pressure on p^*_δ according to (28). Hence, the equilibrium deferred annuity price decreases. Facing a lower deferred annuity price, annuitants buy more deferred annuities in Period 0 and thus are less likely to purchase immediate annuities in Period 1. Therefore, a smaller proportion of annuitants with higher value of θ buy immediate annuities, leading to a higher equilibrium immediate annuity price p^*_α.

[Insert Table 3 here.]

In terms of welfare changes, we consider the proportion of annuitants of each gender group who are adversely affected by the policy, $H(\theta^i_l|i) + \left[1 - H(\theta^i_h|i)\right]$, where $H(\theta|i) = \int_{\theta_L}^{\theta} h(x|i)dx$ is the cumulative density function of gender-i annuitants' survival probabilities, and θ^i_l and θ^i_h are the values of θ such that a gender-i annuitant's utility levels are the same before and after the two policy interventions.[30] It can be concluded from Table 3 that $H(\theta^i_l|i) + \left[1 - H(\theta^i_h|i)\right]$ decreases with g. We also consider the percentage change of aggregate equivalent wealth from actual wealth, $\frac{EW^i(gn\&da) - w^i}{w^i}$, where $EW^i\,(gn\&da)$ is defined in (39). Contrasting to the proportion of annuitants who lose, both extensive and intensive margins of changes in annuitants' welfare are captured by the aggregate equivalent wealth term. We notice from Figure 3 that both the loss of poor-health annuitants as well as the gain of annuitants with average health levels increase when g rises. Since there are more annuitants with average health levels (under the assumption of normal distribution), the welfare of either gender group measured by $\frac{EW^i(gn\&da) - w^i}{w^i}$ increases. Combining the cheaper deferred annuity price and the higher purchased levels of deferred annuities when g increases, it is not surprising to see that both gender groups benefit from a higher gender gap in wealth.

[Insert Figure 3 here.]

[30] For gender-i annuitants, there are either 0, 1 or 2 levels of $\theta \in [\theta_L, \theta_H]$ such that $U^{i*}_\theta\left(w^i, p^*_\delta, p^*_\alpha\right) = \widehat{U}^{i*}_\theta\left(w^i, \widehat{p}^{i*}\right)$. If there is only one level of θ satisfying this condition such that $U^{i*}_\theta\left(w^i, p^*_\delta, p^*_\alpha\right) - \widehat{U}^{i*}_\theta\left(w^i, \widehat{p}^{i*}\right)$ increases from some negative values to become positive, the level is defined as θ^i_l. If there are two levels of θ satisfying this condition, the higher level is defined as θ^i_h.

Next, we analyze the role of the gender gap in health. Compared with gender gap in wealth, there are more ways to represent different gender gaps in health. Our choices are as follows. The baseline case is represented by the gender difference in the mean of the survival probability, $\overline{\theta}^f - \overline{\theta}^m = 0.5 - 0.4 = 0.1$. We also consider three other cases, given in Table 4, such that the changes in $\overline{\theta}^f$ and $\overline{\theta}^m$ are equal and opposite. In these three cases, $\overline{\theta}^f - \overline{\theta}^m = 0$, 0.05 and 0.15, respectively. When there is no gender gap in health with $\overline{\theta}^f = \overline{\theta}^m = 0.45$, only the deferred annuity market exists with the equilibrium price equals to the actuarially fair price, $p^*_\delta = \frac{1}{2}(\overline{\theta}^f + \overline{\theta}^m)$. The results are consistent with Brugiavini (1993) that the immediate annuity market is crowded out completely when all annuitants are identical in health levels in Period 0. When there is gender gap in health, deferred and immediate annuity markets coexist. As shown in Table 4, both θ^f_α and θ^m_α decrease with the increasing health gender gap, indicating that more annuitants buy immediate annuities. Hence, the severity of adverse selection is reduced and the equilibrium price of immediate annuities is lower when the gender gap in health is larger.

[Insert Table 4 here.]

In terms of welfare change, the change in $\widehat{p}^{i*} - p^*_\delta$ is a crucial variable. When the average survival probability for gender i annuitants ($\overline{\theta}^i$) changes, both $\widehat{p}^{i*}$ and p^*_δ changes. Because the gender-neutral price p^*_δ is a weighted average of $\overline{\theta}^f$ and $\overline{\theta}^m$, while the gender-based price $\widehat{p}^{i*}$ depends only on the average survival probability of gender-i annuitants ($\overline{\theta}^i$) and not on that of the other gender, an increase in $\overline{\theta}^f$ leads to an increase in $\widehat{p}^{f*} - p^*_\delta$ and a decrease in $\overline{\theta}^m$ leads to a decrease in $\widehat{p}^{m*} - p^*_\delta$. Thus, $\widehat{p}^{f*} - p^*_\delta$ increases and $\widehat{p}^{m*} - p^*_\delta$ decreases in $\overline{\theta}^f - \overline{\theta}^m$, as observed in Table 4. As a result, more female annuitants benefit from the introduction of deferred annuities and the value of $H(\theta^f_l|f) + (1 - H(\theta^f_h|f))$ decreases. The welfare of the female group, measured by $\frac{EW^f(gn\&da) - w^f}{w^f}$, also increases. On the other hand, when the gender gap in health increases, the proportion of male annuitants who are adversely affected by the policy, $H(\theta^m_l|m) + (1 - H(\theta^m_h|m))$, increases, and the welfare of the male group, $\frac{EW^m(gn\&da) - w^m}{w^m}$, decreases.

There is an interesting comparison regarding the role of the two gender gaps. When there is a larger gender gap in wealth, the male group has a larger share of purchased deferred annuities, leading to a lower price of deferred annuities. Thus, introducing deferred annuities under gender-neutral pricing in an economy with a larger gender gap in wealth benefits more annuitants of both gender groups. On the other hand, in an economy with a larger gender gap in health, the absolute difference of p^*_δ (the deferred annuity price) and

$\widehat{p}^{i*}$ (the immediate annuity price under gender-based pricing when deferred annuities are not available) increases for female annuitants but decreases for male annuitants. As a result, when there is a higher gender gap in health, introducing deferred annuities have different effects on the two gender groups, with the female group benefitting more but the male group benefitting less (or even losing when the gender gap in health is larger than some threshold level).

Based on various cases in Figure 3, we also have an observation about the two adverse effects (over-purchase and intensified distortion effects) faced by the annuitants. We notice that the over-purchase effect affects a larger proportion of annuitants of both gender groups, when compared with the intensified distortion effect. One reason of this asymmetry is that while both good-health and poor-health annuitants are adversely affected, the good-health group at least can still satisfy their residual demands by purchasing the immediate annuity even though it is quite expensive, but poor-health annuitants are not able to do anything regarding their lock-in positions from over-purchasing in the past.

7 Conclusion

Two global trends are relevant to our study. First, many countries emphasize gender equality and ban the use of gender-based pricing. Second, facing rapid population aging, many countries have adopted the defined-contribution pension system, and thus retirees have to bear more responsibility in insuring against longevity risk. We study appropriate annuity policies in this environment. When gender-neutral pricing is adopted in the annuity market, the degree of heterogeneity in the annuitants' health-related characteristics generally increases, since women have higher life expectancy than men. This leads to the well-known result that the severity of adverse selection in the annuity market increases. We consider an additional policy of introducing deferred annuities. Since the health levels of annuitants are less heterogeneous at a younger age when deferred annuities are offered, this policy has the potential of exerting downward pressure on the severity of adverse selection in the annuity market.

We study the effects of introducing deferred annuities under gender-neutral pricing in a two-gender version of the model used by Brugiavini (1993), and obtain several interesting results. Regarding the equilibrium annuity prices, we find in Proposition 2 that the equilibrium deferred annuity price may be lower than the average survival probability of the whole population, because male annuitants (the low-risk group) buy more annuities than female annuitants (the high-risk group) if the effect of gender gap in wealth is stronger than that of gender gap in health, leading to advantageous selection in the deferred annuity market. We also find that, according to Proposition 3, the equilibrium deferred annuity price is always lower than the immediate annuity price for female annuitants under gender-based pricing before deferred annuities are introduced (i.e., $p_\delta^* < \widehat{p}^{f*}$), but may be higher or lower than the corresponding price for male annuitants. The results about equilibrium annuity prices provide the foundation in understanding the affects on annuitants' welfare.

Regarding annuitants' welfare, we find that offering deferred annuities under gender-neutral pricing benefits most women but does not adversely affect too many men. In the baseline case of our numerical analysis, the policy changes benefit 77% of female annuitants and adversely affect only 35% of male annuitants. Second, in terms of comparison within a gender group, the policy benefits annuitants with average health, but adversely affects those on either end. The adverse effects are not symmetric, with poor-health annuitants suffering heavily.

While the above results could provide useful information to policy-makers in designing appropriate annuity policies, further studies are required. In particular, it is observed that poor-health annuitants are more seriously harmed by the introduction of deferred annuities under gender-neutral pricing. A possible follow-up study is to search for appropriate solutions to alleviate the adverse effect on poor-health annuitants of both gender groups, without substantially reducing the benefits of other annuitants.

8 Appendix

Proofs of Propositions 1 to 3 are given in Appendices A to C respectively.

8.1 Appendix A: Proof of Proposition 1

Condition (7) implies that the wealth elasticity of consumption at either period is one. When only immediate annuities are available, $c_{2\theta}^i = \widehat{\alpha}_\theta^{i*}$ according to (6), implying that the wealth elasticity of annuity purchase is one. Therefore, the optimal annuity purchases by men and those by women are related by

$$\widehat{\alpha}_\theta^{m*}(\widehat{p}, w^m) = g\widehat{\alpha}_\theta^{f*}(\widehat{p}, w^f). \tag{A1}$$

Based on (A1), we define

$$J(\widehat{p})=\frac{\int_{\theta_L}^{\theta_H}\theta\widehat{\alpha}_\theta^{f*}\left(\widehat{p},w^f\right)\left[h(\theta\,|f)+gh(\theta\,|m)\right]d\theta}{\int_{\theta_L}^{\theta_H}\widehat{\alpha}_\theta^{f*}\left(\widehat{p},w^f\right)\left[h(\theta\,|f)+gh(\theta\,|m)\right]d\theta} \tag{A2}$$

as a function of $\widehat{p}$. According to (19), (A1) and (A2), the equilibrium price ($\widehat{p}^*$) of an immediate annuity under gender-neutral pricing is defined by the fixed point of function $J(\widehat{p})$:

$$J(\widehat{p}^*)=\widehat{p}^*. \tag{A3}$$

We focus on the interval of $\widehat{p}\in\left[\theta_L,\widehat{p}^{f*}\right]$. First, consider $J(\widehat{p})$ when $\widehat{p}=\theta_L$. It is straightforward to show

$$J(\theta_L)=\frac{\int_{\theta_L}^{\theta_H}\theta\widehat{\alpha}_\theta^{f*}\left(\theta_L,w^f\right)\left[h(\theta\,|f)+gh(\theta\,|m)\right]d\theta}{\int_{\theta_L}^{\theta_H}\widehat{\alpha}_\theta^{f*}\left(\theta_L,w^f\right)\left[h(\theta\,|f)+gh(\theta\,|m)\right]d\theta}>\theta_L. \tag{A4}$$

Second, consider $J(\widehat{p})$ when $\widehat{p}=\widehat{p}^{f*}$. Based on the MLR property assumed in (1), the ratio $\frac{h(\theta|f)+gh(\theta|m)}{h(\theta|f)}$ strictly decreases with $\theta\in[\theta_L,\theta_H]$. Thus, the following Chebyshev's Sum

$$\int_{\theta_L}^{\theta_H}\int_{\theta_L}^{\theta_H}(x-y)\left[\frac{h(x|f)+gh(x|m)}{h(x|f)}-\frac{h(y|f)+gh(y|m)}{h(y|f)}\right]$$
$$\widehat{\alpha}_x^{f*}(\widehat{p}^{f*},w^f)\widehat{\alpha}_y^{f*}(\widehat{p}^{f*},w^f)h(x|f)h(y|f)dxdy \tag{A5}$$

is negative, where x and y are two arbitrary indexes of $\theta\in[\theta_L,\theta_H]$. This leads to[31]

$$J(\widehat{p}^{f*};w^f)=\frac{\int_{\theta_L}^{\theta_H}\theta\widehat{\alpha}_\theta^{f*}\left(\widehat{p}^{f*},w^f\right)\left[h(\theta\,|f)+gh(\theta\,|m)\right]d\theta}{\int_{\theta_L}^{\theta_H}\widehat{\alpha}_\theta^{f*}\left(\widehat{p}^{f*},w^f\right)\left[h(\theta\,|f)+gh(\theta\,|m)\right]d\theta}$$

[31] Expanding (A5) and simplifying lead to

$$\left[\int_{\theta_L}^{\theta_H}x\widehat{\alpha}_x^{f*}(\widehat{p}^{f*},w^f)\left[h(x|f)+gh(x|m)\right]dx\right]\left[\int_{\theta_L}^{\theta_H}\widehat{\alpha}_y^{f*}(\widehat{p}^{f*},w^f)h(y|f)dy\right]$$
$$-\left[\int_{\theta_L}^{\theta_H}x\widehat{\alpha}_x^{f*}(\widehat{p}^{f*},w^f)h(x|f)dx\right]\left[\int_{\theta_L}^{\theta_H}\widehat{\alpha}_y^{f*}(\widehat{p}^{f*},w^f)\left[h(y|f)+gh(y|m)\right]dy\right]$$
$$-\left[\int_{\theta_L}^{\theta_H}\widehat{\alpha}_x^{f*}(\widehat{p}^{f*},w^f)\left[h(x|f)+gh(x|m)\right]dx\right]\left[\int_{\theta_L}^{\theta_H}y\widehat{\alpha}_y^{f*}(\widehat{p}^{f*},w^f)h(y|f)dy\right]$$
$$+\left[\int_{\theta_L}^{\theta_H}\widehat{\alpha}_x^{f*}(\widehat{p}^{f*},w^f)h(x|f)dx\right]\left[\int_{\theta_L}^{\theta_H}y\widehat{\alpha}_y^{f*}(\widehat{p}^{f*},w^f)\left[h(y|f)+gh(y|m)\right]dy\right]<0.$$

$$< \frac{\int_{\theta_L}^{\theta_H} \theta \widehat{\alpha}_\theta^{f*}\left(\widehat{p}^{f*}, w^f\right) h(\theta\,|f\,)d\theta}{\int_{\theta_L}^{\theta_H} \widehat{\alpha}_\theta^{f*}\left(\widehat{p}^{f*}, w^f\right) h(\theta\,|f\,)d\theta} = \widehat{p}^{f*}. \tag{A6}$$

Combining (A4), (A6), and the continuity of function $J(\widehat{p})$, we conclude that there exists an equilibrium value of $\widehat{p}^*$ such that $\theta_L < J(\widehat{p}^*) = \widehat{p}^* < \widehat{p}^{f*}$.[32] Hence, we conclude that $\widehat{p}^* < \widehat{p}^{f*}$. Graphically, $\widehat{p}^*$ is determined by the intersection of the $J(\widehat{p})$ function and the 45-degree line, as shown in Panel A of Figure A1.

[Insert Figure A1 here.]

Next, in order to compare $\widehat{p}^{m*}$ with $\widehat{p}^*$, we define

$$K(\widehat{p}) = \frac{\int_{\theta_L}^{\theta_H} \theta \widehat{\alpha}_\theta^{m*}\left(\widehat{p}, w^m\right)\left[\frac{1}{g}h(\theta\,|f\,) + h(\theta\,|m)\right] d\theta}{\int_{\theta_L}^{\theta_H} \widehat{\alpha}_\theta^{m*}\left(\widehat{p}, w^m\right)\left[\frac{1}{g}h(\theta\,|f\,) + h(\theta\,|m)\right] d\theta} \tag{A7}$$

as a function of $\widehat{p}$. According to (19), (A1) and (A7), the equilibrium price ($\widehat{p}^*$) of an immediate annuity under gender-neutral pricing is defined by the fixed point of function $K(\widehat{p})$:

$$K(\widehat{p}^*) = \widehat{p}^*. \tag{A8}$$

In the Online Appendix, we show that there exists an equilibrium value of $\widehat{p}^* \in [\widehat{p}^{m*}, \theta_H]$ such that $\widehat{p}^{m*} < K(\widehat{p}^*) = \widehat{p}^* < \theta_H$. (The proof is based on a method similar to that used to prove $\widehat{p}^* < \widehat{p}^{f*}$.) See Panel B of Figure A1.

Together with $\widehat{p}^* < \widehat{p}^{f*}$ shown above, we obtain (22). This proves part (a).

After replacing the arbitrary indices of x and y by θ, we obtain

$$\left[\int_{\theta_L}^{\theta_H} \theta \widehat{\alpha}_\theta^{f*}\left(\widehat{p}^{f*}, w^f\right)\left[h(\theta\,|f\,) + gh(\theta\,|m)\right] d\theta\right]\left[\int_{\theta_L}^{\theta_H} \widehat{\alpha}_\theta^{f*}\left(\widehat{p}^{f*}, w^f\right) h(\theta\,|f\,)d\theta\right]$$

$$< \left[\int_{\theta_L}^{\theta_H} \theta \widehat{\alpha}_\theta^{f*}\left(\widehat{p}^{f*}, w^f\right) h(\theta\,|f\,)d\theta\right]\left[\int_{\theta_L}^{\theta_H} \widehat{\alpha}_\theta^{f*}\left(\widehat{p}^{f*}, w^f\right)\left[h(\theta\,|f\,) + gh(\theta\,|m)\right] d\theta\right],$$

which leads to the inequality in (A6). Using (16) and (A3), we obtain (A6).

[32] According to the computational analysis in Section 6, all equilibrium annuity prices defined in (14), (15), (16) and (19) are unique. However, we know from the results in Abel (1986, p. 1086) and Villeneuve (2003, p. 534) that when the annuitants' choices and annuity price are mutually dependent, the uniqueness of equilibrium is not guaranteed generally. In this paper, we simply assume that the equilibrium annuity prices are unique, which is consistent with the computational results.

Using (19), (21) and $cov\left(\theta, \widehat{\alpha}_\theta^{i*}\left(\widehat{p}^*, w^i\right)\right) = \int_{\theta_L}^{\theta_H} \theta \widehat{\alpha}_\theta^{i*}\left(\widehat{p}^*, w^i\right) h(\theta|i) d\theta - \overline{\theta}^i E\left(\widehat{\alpha}_\theta^{i*}\left(\widehat{p}^*, w^i\right)\right)$, we obtain

$$\widehat{p}^* - \frac{1}{2}(\overline{\theta}^f + \overline{\theta}^m) = \frac{cov\left(\theta, \widehat{\alpha}_\theta^{f*}\left(\widehat{p}^*, w^f\right)\right) + cov\left(\theta, \widehat{\alpha}_\theta^{m*}\left(\widehat{p}^*, w^m\right)\right)}{E(\widehat{\alpha}_\theta^{f*}(\widehat{p}^*, w^f)) + E(\widehat{\alpha}_\theta^{m*}(\widehat{p}^*, w^m))}$$

$$+ \frac{\overline{\theta}^f E(\widehat{\alpha}_\theta^{f*}(\widehat{p}^*, w^f)) + \overline{\theta}^m E(\widehat{\alpha}_\theta^{m*}(\widehat{p}^*, w^m))}{E(\widehat{\alpha}_\theta^{f*}(\widehat{p}^*, w^f)) + E(\widehat{\alpha}_\theta^{m*}(\widehat{p}^*, w^m))} - \frac{1}{2}(\overline{\theta}^f + \overline{\theta}^m). \tag{A9}$$

After using $\widehat{\beta}^f$ in (20) and simplifying, (A9) leads to (23). This proves part (b). ■

8.2 Appendix B: Proof of Proposition 2

It is straightforward to show that an equilibrium of this economy exists, with p_α^* in the interval $[\theta_L, \theta_H]$ and p_δ^* in the interval $\left[\overline{\theta}^m, \overline{\theta}^f\right]$. The intuition is that setting a high price (such as θ_H) will lead to budget surplus for annuity providers and setting a low price (such as θ_L) leads to budget deficit. By continuity of the annuity provider's profit functions, there exists an equilibrium satisfying the two zero-profit conditions (14) and (15).

We prove part (a) by contradiction.

Suppose no annuitant buys the immediate annuity (i.e., $\alpha_\theta^{i*} = 0$ for all θ and i). Then all annuitants will buy deferred annuities (i.e., $\delta^{i*} > 0$) to insure against longevity risk. As a result. δ^{i*} satisfies the first-order condition (13). In particular, (13) of male annuitants becomes

$$\int_{\theta_L}^{\theta_H} \frac{p_\delta^*}{1+r} u'\left((1+r)\, w^m - \frac{p_\delta^*}{1+r}\delta^{m*}\right) h(\theta\,|m)\, d\theta = \int_{\theta_L}^{\theta_H} \frac{\theta}{1+\rho} u'\left(\delta^{m*}\right) h(\theta\,|m) d\theta.$$

Since the integrand on the LHS of the above equation is independent of θ, we obtain

$$\frac{p_\delta^*}{1+r} u'\left((1+r)\, w^m - \frac{p_\delta^*}{1+r}\delta^{m*}\right) = \frac{\overline{\theta}^m}{1+\rho} u'\left(\delta^{m*}\right). \tag{A10}$$

Based on (14) and (15), the following two results are useful:

$$p_\alpha^* \leq \frac{\int_{\theta_L}^{\theta_H} \theta_H \left[\alpha_\theta^{f*} h\left(\theta\,|f\right) + \alpha_\theta^{m*} h\left(\theta\,|m\right)\right] d\theta}{\int_{\theta_L}^{\theta_H} \left[\alpha_\theta^{f*} h\left(\theta\,|f\right) + \alpha_\theta^{m*} h\left(\theta\,|m\right)\right] d\theta} = \theta_H, \tag{A11}$$

and

$$p_\delta^* = \frac{\delta^{f*}\overline{\theta}^f + \delta^{m*}\overline{\theta}^m}{\delta^{f*} + \delta^{m*}} > \overline{\theta}^m. \tag{A12}$$

Combining (A10) to (A12), we obtain[33]

$$\frac{p_\alpha^*}{1+r}u'\left((1+r)\,w^m - \frac{p_\delta^*}{1+r}\delta^{m*}\right) < \frac{\theta_H}{1+\rho}u'\left(\delta^{m*}\right), \tag{A13}$$

which indicates that the marginal cost of buying the first unit of immediate annuity is smaller than its marginal benefit for male annuitants with $\theta = \theta_H$. This result contradicts the supposition that $\alpha_\theta^{i*} = 0$ for all θ and i.

Suppose no annuitant buys deferred annuities ($\delta^{f*} = \delta^{m*} = 0$). In this case, each annuitant will buy immediate annuities ($\alpha_\theta^{i*} > 0$), and it is easy to show that $p_\alpha^* = \widehat{p}^*$, where $\widehat{p}^*$ is defined in (19). Because the possibility of $p_\delta^* \geq p_\alpha^* = \widehat{p}^*$ is ruled out by (26), we only need to consider whether the combination of $\delta^{f*} = \delta^{m*} = 0$ and $p_\delta^* < p_\alpha^* = \widehat{p}^*$ is possible.

Since all immediate annuity purchases (α_θ^{i*}) are interior solutions when $\delta^{i*} = 0$ for both gender groups, the first-order condition (10) becomes

$$\frac{p_\alpha^*}{1+r}u'\left((1+r)\,w^i - \frac{p_\alpha^*}{1+r}\alpha_\theta^{i*}\right) = \frac{\theta}{1+\rho}u'\left(\alpha_\theta^{i*}\right).$$

Multiply both sides of this equation by $h(\theta\,|i)$ and integrate with respect to θ. Combining the resulting equation with $p_\delta^* < p_\alpha^* = \widehat{p}^*$ leads to

$$\int_{\theta_L}^{\theta_H} \frac{p_\delta^*}{1+r}u'\left((1+r)\,w - \frac{p_\delta^*}{1+r}\alpha_\theta^{i*}\right) h(\theta\,|i)\,d\theta$$

$$< \int_{\theta_L}^{\theta_H} \frac{p_\alpha^*}{1+r}u'\left((1+r)\,w^i - \frac{p_\alpha^*}{1+r}\alpha_\theta^{i*}\right) h(\theta\,|i)\,d\theta = \int_{\theta_L}^{\theta_H} \frac{\theta}{1+\rho}u'\left(\alpha_\theta^{i*}\right) h(\theta\,|i)\,d\theta. \tag{A14}$$

We see from (A14) that for annuitants of either gender, the marginal cost of buying the first unit of deferred annuity is lower than the marginal benefit. Thus, condition (A14) is inconsistent with the supposition of no deferred annuity transaction ($\delta^{f*} = \delta^{m*} = 0$). This proves part (a).

Both deferred and immediate annuities help the annuitants insure against longevity risk, but the immediate annuities are offered at a later time when

[33] Using (A10), (A12) and (A11) sequentially, we obtain

$$\frac{p_\alpha^*}{1+r}u'\left((1+r)\,w^m - \frac{p_\delta^*}{1+r}\delta^{m*}\right) = \left(\frac{p_\alpha^*}{p_\delta^*}\right)\frac{\overline{\theta}^m}{1+\rho}u'\left(\delta^{m*}\right) < \left(\frac{p_\alpha^*}{p_\delta^*}\right)\frac{p_\delta^*}{1+\rho}u'\left(\delta^{m*}\right) \leq \frac{\theta_H}{1+\rho}u'\left(\delta^{m*}\right).$$

private health information is revealed. If the deferred annuity is not cheaper than the immediate annuity (i.e., $p_\delta^* \geq p_\alpha^*$), the annuitants would find it optimal to wait to receive health information in Period 1 and then make annuitization purchases. These decisions imply that there is no purchase of the deferred annuity in Period 0. Thus, coexistence of deferred and immediate annuity transactions is incompatible with $p_\delta^* \geq p_\alpha^*$. Therefore, (27) in part (b) holds.

It is straightforward to use the formula of p_δ^* in (15) to obtain (28) in part (c).

This proves Proposition 2. ■

8.3 Appendix C: Proof of Proposition 3

According to (17), $\widehat{p}^{f*} > \overline{\theta}^f$, because $\widehat{\alpha}_\theta^{i*}(\widehat{p}^{i*}, w^i)$ and θ are positively correlated. According to (15), p_δ^* is a weighted average of $\overline{\theta}^f$ and $\overline{\theta}^m$. Since $\overline{\theta}^f > \overline{\theta}^m$, we have

$$p_\delta^* \leq \overline{\theta}^f. \tag{A15}$$

Combining these two results leads to (29).

If condition (31) holds, then combining it with (17) for male annuitants and (A15) leads to

$$\widehat{p}^{m*} - \overline{\theta}^m = \frac{cov\left(\theta, \widehat{\alpha}_\theta^{m*}\left(\widehat{p}^{m*}, w^m\right)\right)}{E(\widehat{\alpha}_\theta^{m*}\left(\widehat{p}^{m*}, w^m\right))} > \overline{\theta}^f - \overline{\theta}^m \geq p_\delta^* - \overline{\theta}^m.$$

Thus, we have (30). ■

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Table 1: Comparison of three annuity markets under gender-neutral pricing

	λ^{wg}	λ^{bg}	$\lambda = p^* - 0.5\left(\overline{\theta}^f + \overline{\theta}^m\right) = \lambda^{wg} + \lambda^{bg}$
Voluntary purchase of immediate annuities (23), before deferred annuities are introduced	Positive	Positive or negative	Less likely to be negative
Voluntary purchase of deferred annuities (28)	0	Positive or negative	More likely to be negative
Mandatory purchase of annuities, with partial waiver: (17) of Lau et al. (2023)	Positive	Negative	More likely to be negative

Table 2: The self-reported probability of living to age 85 (θ)

	Observations	Mean (%)	Std. Dev (%)
Total	20,873	43.536	32.077
Women	11,212	46.274	31.873
Men	9,661	40.358	32.023

Table 3: The role of gender gap in wealth

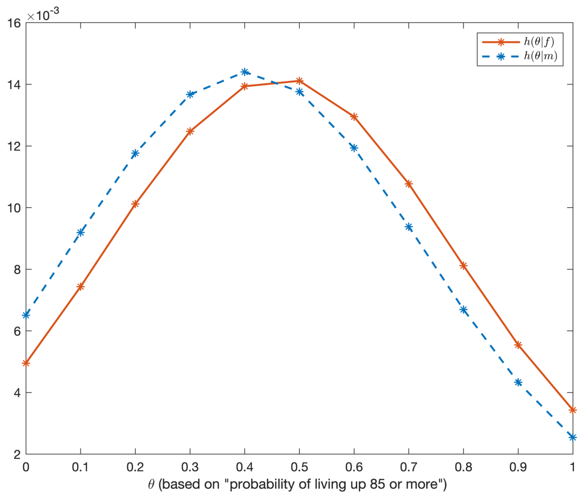
g	1 (no gap)	1.2	1.44	2
δ^{f*}	111.25	113.17	115.16	118.83
δ^{m*}	78.49	96.21	117.93	169.99
$\delta^{f*} - \delta^{m*}$	32.76	16.96	−2.77	−51.16
θ_α^f	0.99	θ_H	θ_H	θ_H
θ_α^m	0.78	0.80	0.82	0.84
p_δ^*	0.4586	0.4540	0.4494	0.4411
p_α^*	0.9092	0.9171	0.9252	0.9391
$H(\theta_l^f\|f)$	23.00%	22.81%	22.63%	22.31%
$H(\theta_l^m\|m)$	31.46%	31.28%	31.06%	30.72%
$1 - H(\theta_h^f\|f)$	0%	0%	0%	0%
$1 - H(\theta_h^m\|m)$	5.11%	4.27%	3.49%	2.27%
$\frac{EW^f(gn\&da) - w^f}{w^f}$	4.84%	5.15%	5.46%	6.05%
$\frac{EW^m(gn\&da) - w^m}{w^m}$	−0.28%	−0.08%	0.13%	0.50%

Table 4: The role of gender gap in health

$\overline{\theta}^f - \overline{\theta}^m$	0 (0.45 − 0.45)	0.05 (0.475 − 0.425)	0.10 (0.5 − 0.4)	0.15 (0.525 − 0.375)
δ^{f*}	98.81	107.89	115.16	119.78
δ^{m*}	142.28	131.67	117.93	100.02
θ_α^f	/	θ_H	θ_H	0.98
θ_α^m	/	0.92	0.82	0.68
p_δ^*	0.4500	0.4475	0.4494	0.4567
p_α^*	/	0.9704	0.9252	0.8539
$\hat{p}^{f*} - p_\delta^*$	0.1646	0.1847	0.1999	0.2092
$\hat{p}^{m*} - p_\delta^*$	0.1646	0.1490	0.1282	0.1011
$H(\theta_l^f \mid f)$	26.88%	24.63%	22.63%	20.86%
$H(\theta_l^m \mid m)$	26.88%	28.90%	31.06%	33.18%
$1 - H(\theta_h^f \mid f)$	0%	0%	0%	0%
$1 - H(\theta_h^m \mid m)$	0%	0%	3.49%	10.08%
$\frac{EW^f(gn\&da) - w^f}{w^f}$	2.52%	4.07%	5.46%	6.54%
$\frac{EW^m(gn\&da) - w^m}{w^m}$	2.52%	1.37%	0.13%	−1.13%

Figure 1: Distributions of survival probability by gender

Penal A: $h(\theta|f)$ and $h(\theta|m)$



Penal B: Likelihood ratio $\frac{h(\theta|f)}{h(\theta|m)}$

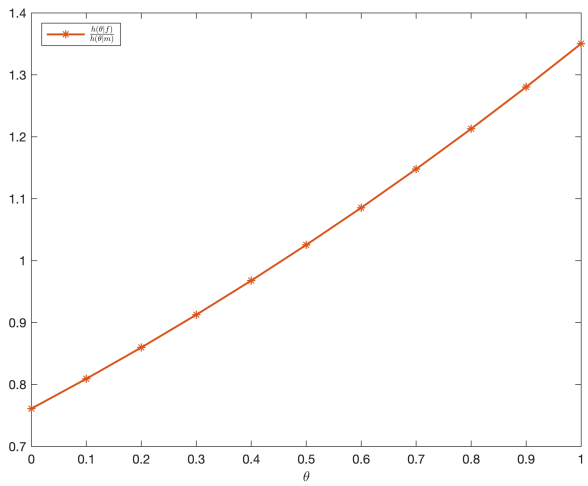


Figure 2: Welfare effects of policy changes

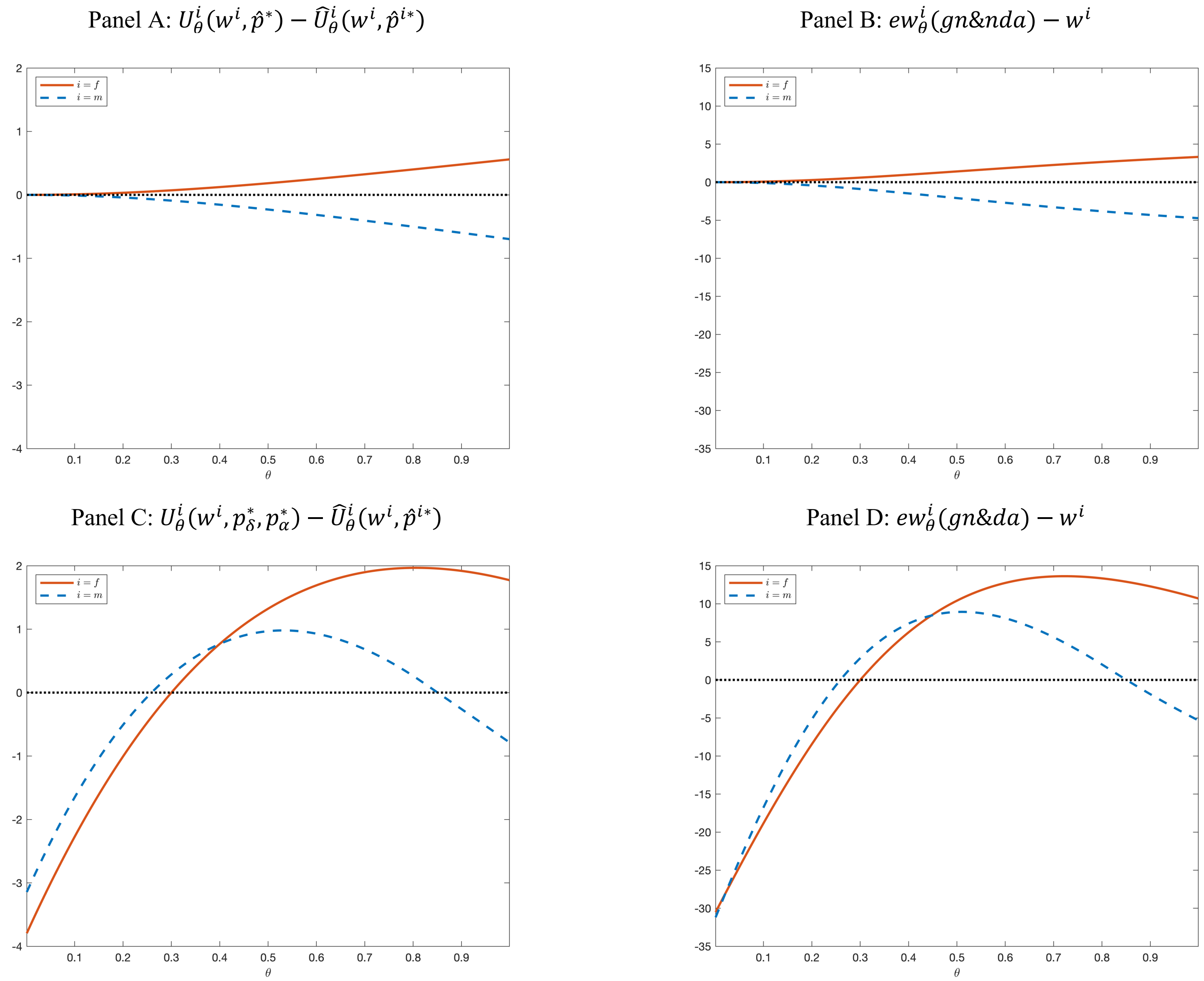


Figure 3: $ew_{\theta}^{i}(gn\&da) - w^{i}$ for various gender gaps in wealth and health

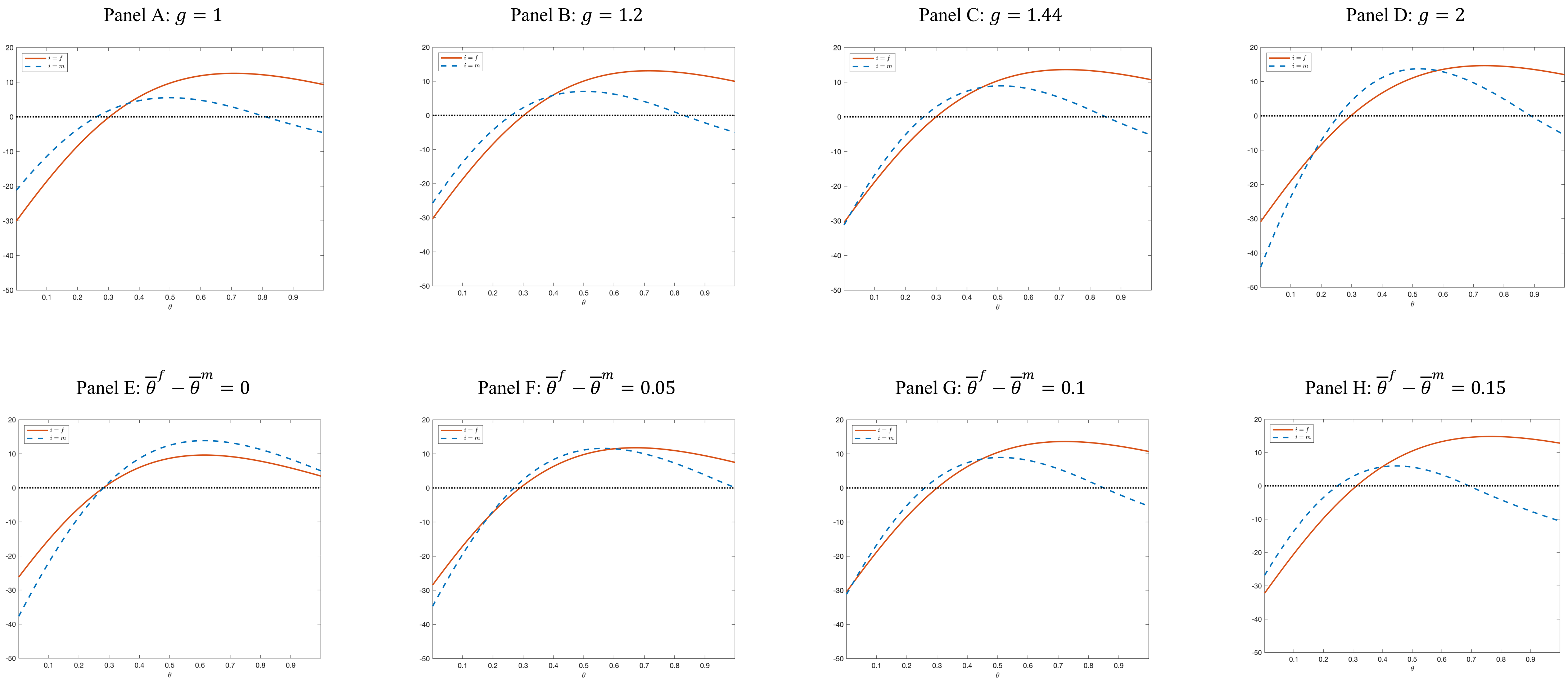


Figure A1: Using functions $J(\hat{p})$ and $K(\hat{p})$ to analyze the equilibrium value $\hat{p}^*$

Panel A: Function $J(\hat{p})$ Panel B: Function $K(\hat{p})$