Banning gender-based pricing in a mandatory annuity program with partial waiver may unintentionally lead to advantageous selection

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Abstract

A standard solution to adverse selection in insurance markets is to mandate all buyers to purchase insurance. This paper revisits this solution in a mandatory annuity program with partial waiver, which is empirically relevant. With the assumptions of positive health-wealth correlation and gender gaps in health and wealth, we obtain two main results. First, under gender-based pricing, adverse selection is not eliminated when annuity purchases are mandatory. Second, based on decomposing the severity of adverse selection into the within-group and between-group effects under gender-neutral pricing, advantageous selection may be present if the between-group effect is stronger than the within-group effect.

Keywords: mandatory annuity program; partial waiver; gender-based pricing; gender-neutral pricing; adverse selection; advantageous selection

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1 Introduction

It is well known that asymmetric information is present in many insurance markets (Akerlof, 1970; Rothschild and Stiglitz, 1976). In this environment, higher-risk buyers choose to purchase larger amounts of the product, leading to adverse selection and market inefficiency. A standard solution to eliminate the inefficiency associated with the positive correlation of the risk type and the purchase amount is to forbid choices by mandating that all buyers purchase insurance.[1]

This paper revisits this solution to adverse selection and finds some interesting results in an economy with a mandatory public annuity program and gender-neutral pricing.[2] These two policy interventions, annuitization mandating and gender-neutral pricing, are currently observed in some countries and likely to be more prevalent in the coming years.

The first policy intervention, mandatory public annuity program, is related to population aging, which is widely observed in many countries.[3] This demographic trend imposes pressure on the retirement income protection system in many developed countries, particularly those adopting the unfunded pay-as-you-go (PAYGO) system where the benefits paid to current pensioners are mainly financed by the contributions from current workers. In order to solve the financial unsustainability problem in many existing pension systems, a lot of governments have undertaken reforms by building up the fully-funded defined-contribution system with Individual Accounts (Mandatory Provident Fund Schemes Authority, 2015, Table 3.1; OECD, 2021, pp. 49-51). In addition, some governments have linked the purchase of annuities to the pensioners' retirement wealth, providing a channel for the pensioners to insure against the risk of outliving their resources. Lau and Zhang (2023) review different public annuity programs in various economies; in particular, mandatory and voluntary public annuity programs are possible options to be adopted. For example, the governments of Denmark, Lithuania, Singapore and Sweden require the pensioners to use their savings in the Individual Accounts to buy public annuities as the mandatory decumulation option. Since we want to analyze the use of mandatory annuity purchase as a solution of

[1] Einav and Finkelstein (2011, p. 120) mention that: "The canonical solution to the inefficiency created by adverse selection is to mandate that everyone purchase insurance."

[2] Public annuities are chosen because when compared with other insurance products, it is more likely to observe mandatory purchase of annuities in a public defined-contribution pension system. Moreover, as elaborated in the next paragraph, this kind of retirement income protection system has become more prevalent in recent years.

[3] According to OECD (2019), the percentage of population aged 65 and above in the OECD countries is projected to increase from 17.4 percent in 2017 to 27.1 percent in 2050.

adverse selection in the annuity market, we focus on the mandatory public annuity programs in which the pensioners do not have the freedom to choose the annuitization amount.[4]

Another policy intervention relevant to our analysis is the use of gender-neutral, instead of gender-based, pricing in the annuity market. Starting from 2012, gender-based pricing has been banned in the insurance sector in the European Union (EU), with gender equality regarded as the fundamental right. It is likely that gender equality will be increasingly emphasized in other societies in the coming years, leading to more use of gender-neutral pricing in the insurance (including annuity) market.

In light of these two major trends, this paper examines the implications of adopting gender-based versus gender-neutral pricing in the mandatory public annuity programs. A key element of our analysis is a mandatory annuity program with partial waiver, which usually arises because of affordability concern. We consider this program, instead of a strict mandatory annuity program in which all pensioners are required to buy the same amount of annuities, because of two reasons. First, the result of eliminating adverse selection under a strict mandatory annuity program is straightforward and well known. Second, while the result about a strict mandatory annuity program is of theoretical interest, it may not be empirically relevant for many countries. In order to have a impactful mandatory public annuity policy, the mandated purchase is usually set at a not-too-low level. As a result, not every pensioner has adequate wealth to fulfil this obligation. In reality, the government is likely to introduce some exemption clauses so that poor pensioners may have some degree of waiver.[5] As shown in subsequent sections, the feature of partial waver turns out to be a major factor determining the key results of this paper.

Taking the above features into consideration, this paper studies a mandatory public annuity program with partial waiver, with either gender-based or gender-neutral pricing. We also assume three commonly-observed features about health and wealth of the male and female pensioners: male pensioners have higher average wealth but lower life expectancy when compared with the

[4]According to Lau and Zhang (2023, Table 1), pensioners may be given some freedom to choose the annuitization amount in some public annuity programs even though they are mainly classified as mandatory, as in Singapore and Sweden. In order to deliver sharp results, this paper considers the version of mandatory public annuity programs in which pensioners are not allowed to choose the annuitization amount. On the other hand, Lau and Zhang (2023) have analyzed public annuity programs in which pensioners choose the annuitization levels.

[5]In Section 3.1, we will elaborate on this point and present evidence of partial wavier in Denmark and Lithuania.

female pensioners, and life expectancy and wealth are positively correlated in either group.

Based on a simple model capturing the relevant elements, we introduce a measure of the severity of adverse selection in public annuities.[6] We then study two major issues. First, we examine whether the inefficiency associated with adverse selection is eliminated in a mandatory public annuity program with partial waiver when gender-based pricing is adopted. Contrary to conventional wisdom, we find that the answer is no. The intuition of the result is due to the interaction of the partial waiver and positive correlation of pensioners' health and wealth. Within each gender group, poorer pensioners tend to be the low-risk type, because health and wealth are positively correlated. The partial waiver essentially reduces the share of low-risk type, leading to a positive value of the severity of adverse selection in each gender group.[7] Second, we examine the effect of banning gender-based pricing on the severity of adverse selection of the annuity market. Previous studies by Hoy (1982), Crocker and Snow (1986), Finkelstein et al. (2009) and Aquilina et al. (2017) suggest that in the presence of asymmetric information, banning the use of categorical discrimination based on observed characteristics (such as gender) would lead to a higher severity of adverse selection. By decomposing the effect of imposing gender-neutral pricing into the within-group and between-group components, we find a new result that the severity of adverse selection may be negative when gender-based pricing is banned in the mandatory public annuity program with partial waiver.[8] Once again,

[6] As will be shown in Sections 3 and 6, this term is important because it is related to pensioners' welfare under the zero-profit condition or to the annuity provider's budget balance if the annuity payout is set at the actuarially fair level. In particular, under the zero-profit condition, the pensioners are adversely affected in the presence of adverse selection because the annuity payout is below the actuarially fair level, but benefit when the severity of adverse selection is negative (and advantageous selection appears) because the annuity payout is above the actuarially fair level.

[7] The term adverse selection refers to the positive correlation of the risk type and the choice variable when information about the risk type is asymmetric, leading to an adverse outcome. For many interesting cases, both elements of *selection* (or choice) and *adverse outcome* are present. In this paper, we focus on a mandatory annuity program in which the pensioners' annuity purchases are mandated. In this environment, one main issue studied is whether the conventional wisdom that "adverse selection is eliminated when choice is disallowed under mandatory annuity purchase" holds or not when gender-based pricing is adopted, even though the pensioners' annuity purchases are mandated (i.e., passive). Instead of adopting "adverse outcome under passive selection," using the term "adverse selection" in the presence of passive selection is simpler and consistent with the standard use in the literature.

[8] As mentioned in the previous footnote, we focus on mandatory annuity programs when the pensioners' annuity purchases are passive. Strictly speaking, "advantageous outcome

the partial waiver is a crucial factor. While it still causes a more severe effect of adverse selection within each gender group when gender-neutral pricing is imposed, the partial waiver leads to an extra between-group effect of a lower share of female pensioners who are poorer but are the high-risk group (with better health on average). Banning gender-based pricing may unintentionally lead to advantageous selection (i.e., a negative value of the severity of adverse selection) when the between-group effect is stronger than the within-group effect.

The remaining sections of this paper are organized as follows. Section 2 provides relevant literature review. Section 3 introduces a simple model based on the observed mandatory public annuity programs and several important ingredients regarding health and wealth characteristics of the pensioners. It also introduces a useful measure of the severity of adverse selection when the public annuity payout level is determined according to the zero-profit condition. Section 4 considers the mandatory annuity program with gender-based pricing, paying particular attention to the severity of adverse selection. Section 5 examines the corresponding issues when gender-neutral pricing is adopted. In Section 6 we assume that the annuity payout is set at the actuarially fair level, and link the budget balance of the annuity provider under this assumption to the severity of adverse selection under the zero-profit assumption. Section 7 provides the concluding remarks.

2 Literature review

This paper is related to two strands of the literature. The first is the literature on the effect of asymmetric information in insurance markets, including that of the annuities. Rothschild and Stiglitz (1976) point out that asymmetric information of risk types in the insurance market generally leads to market inefficiency. In the annuity market, the buyers differ in their health conditions which are usually unknown to the annuity providers. Finkelstein and Poterba (2002, 2004) show that adverse selection is present in the United Kingdom (UK) annuity market. Interestingly, Cawley and Philipson (1999), Finkelstein and McGarry (2006) and Fang et al. (2008) have examined various insurance markets and found empirical evidence more consistent with the phenomenon of advantageous selection, which has been discussed in earlier theoretical models of Hemenway (1990) and de Meza and Webb (2001). Motivated by these results in the literature, this paper examines the pres-

under passive selection" is a more accurate description of this result, but we keep using the term "advantageous selection" in this environment because it would better connect our results with the literature.

ence of adverse or advantageous selection associated with mandatory annuity purchase to insure against longevity risk. Besides including the heterogenous risk types (in survival probability) relevant to the annuity market, this paper also incorporates the heterogeneity of wealth which is particularly important in the mandatory public annuity programs in many countries. This factor turns out to be one of the important factors affecting the results in this paper.

Second, it is related to the literature about introducing public annuities to help pensioners hedge against longevity risk. It is well known that only a small percentage of consumers purchase annuities in the private market, contradictory to the sharp prediction in Yaari (1965) that consumers would benefit by annuitizing all their wealth under some assumptions. This discrepancy generates interest among researchers to study the annuity puzzle (Benartzi et al., 2011). One factor offered to explain the high price of the private annuity (and thus the small amount of sales) is the high cost of private annuity providers (Friedman and Warshawsky, 1990). On the other hand, Diamond (2004) suggests that the government is able to provide the annuity product with lower cost when compared to the private market. Motivated by the observed practices of public annuities in different economies, Lau and Zhang (2023) study policy design questions related to voluntary public annuity programs. Since some countries providing public annuities adopt the mandatory system instead of the voluntary type, this paper studies the mandatory public annuity program. We pay careful attention to the details of this program such as the exemption clause, and find that this clause is important in generating unexpected results, particularly when it is combined with another policy intervention of banning gender-based pricing.

3 Mandatory annuity programs in the presence of health-wealth correlation and gender differences

In Section 3.1, we use a simple analytical model to capture some essential features of mandatory annuity programs. In Section 3.2, we discuss three main assumptions about health and wealth characteristics of male and female pensioners. In Section 3.3, we introduce an important measure of the severity of adverse selection, which is useful for subsequent analysis.

3.1 Mandatory annuity programs with partial waiver

With the trend of population aging, several governments build up more financially sustainable defined-contribution schemes to complement or even replace the conventional unfunded PAYGO pension system. Furthermore, to insure against longevity risk for the pensioners, several governments (including Denmark, Lithuania, Singapore and Sweden) provide the annuity product and require the pensioners to use their accumulated contributions to purchase the public annuities at retirement.

Since the main idea in this paper is most clearly illustrated in a mandatory public annuity program with no choice element, we consider a model in which all pensioners are required to purchase the annuity provided by the government. However, instead of assuming a strict version of the mandatory annuity program in which all pensioners are required to purchase the same amount of public annuity, we consider an empirically relevant mandatory public annuity program with a waiver clause to some pensioners, as in Denmark and Lithuania. A possible reason is that in a strict mandatory public annuity program, especially if it is one in which the mandated level is set at a not-too-low level to be impactful, then some pensioners with low retirement wealth may have difficulty in attaining this level. A compromise in this situation is to allow pensioners with low wealth level some exemption, perhaps because of affordability concern. We label this arrangement as a mandatory annuity program with partial waiver.

We use a two-period framework (as in Abel, 1986; Villeneuve, 2003) to represent the mandatory public annuity program with partial waiver. It is assumed that every pensioner in the model survives at least one period, but it is uncertain whether a particular pensioner can survive to the second period. Moreover, there is heterogeneity in the survival probability among the pensioners. The pensioner's health level is denoted by $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ (with $0 \leq \underline{\theta} < \overline{\theta} \leq 1$), the probability of surviving to the Period 2.

To help the pensioners insure against longevity risk, the government introduces a public annuity contract with survival-contingent payment only. In Period 1, the pensioners purchase the public annuity. Those pensioners who survive to Period 2 will receive the annuity payment. To represent the partial waiver component in an analytically convenient way, we assume that the level of public annuity purchase is determined according to

$$\alpha(w) = \min\{\gamma w, M\}, \tag{1}$$

where $\alpha(w)$ is defined as the mandated level of public annuity purchase for a pensioner with pension wealth w (with $\underline{w} \leq w \leq \overline{w}$). All pensioners, except those with low pension wealth, are required to purchase the same amount (M units) of the public annuity, according to (1). Pensioners with wealth below the threshold level $\frac{M}{\gamma}$ are exempted partially and need only to purchase γw units of the public annuity, where γ ($0 < \gamma \leq 1$) is the percentage of the pensioner's retirement wealth that is required to purchase public annuities.[9] Note that under this mandatory public annuity program, the amounts of public annuity purchase of pensioners directly depend on their pension wealth w but not their survival probabilities θ. The public annuity purchase $\alpha(w)$ is plotted as a function of the individual's pension wealth w in Panel A of Figure 1.

[Insert Figure 1 here.]

The features of mandatory public annuity purchase and partial waiver are consistent with some observed practices in various countries. For example, pensioners in Lithuania with the level of pension wealth equal to or above Euro 10,000 in their Individual Accounts of the Tier II pension fund are required to join the public annuity program.[10] Moreover, compared with the mandated amount of public annuity purchase by pensioners with pension wealth equal to or higher than Euro 60,000, pensioners with a lower level of pension wealth from Euro 10,000 to Euro 60,000 have partial exemption by buying the public annuity with all their wealth in the Tier II pension fund.[11] Besides Lithuania, we also observe the features of mandatory annuitization and partial waiver in the ATP (Danish Labour Market Supplementary Pension) scheme in Denmark.[12] Members of this scheme who work full time (117

[9] We use the linearity assumption of γw when $w < \frac{M}{\gamma}$ in (1) for simplicity. It can be shown that our results also hold if we replace the linearity assumption by a more general specification of an increasing function of w for the pensioners' public annuity purchases when w is below some threshold level.

[10] Information about the Lithuanian public annuity plan can be found in the website: https://www.sodra.lt/en/benefits/pension-annuity-payment. Note that the Tier II pension fund is just one aspect of the pension system in Lithuania.

[11] Applying (1) to the Lithuanian program, the value of M corresponds to Euro 60,000. Note that some choices in the amount of public annuity purchase are allowed for low-wealth pensioners (with account balance below Euro 10,000) or high-wealth pensioners (with account balance above Euro 60,000) in Lithuania. We do not model these aspects because we want to keep the model relatively simple. We also assume that there is only one type of public annuity (or one type of public annuity for each gender under gender-based pricing) in our model.

[12] ATP was established by the Danish Parliament in 1964. Information about the statutory, defined-contribution ATP Livslang Pension (Lifelong Pension) scheme can be found in relevant websites (https://www.atp.dk/en/atp-lifelong-pension; https://lifeindenmark.borger.dk/pension/atp-livslang-pension/atp-contribution-rates-private-sector). Similar to Lithuania, this defined-contribution scheme is only one aspect of the pension system in Denmark.

hours or more per month) are required to contribute a fixed amount to their Individual Accounts, irrespective of the actual number of hours of work. On the other hand, those who work less than 117 hours per month contribute smaller amounts corresponding to the actual hours of work. Since a member's amount of mandatory annuitization is equal to the value of one's previous contributions, pensioners who had worked part time and contributed less than the full-time standard in the past are "partially waived" in the purchase of annuities provided by the ATP. However, some aspects of our model are different from the diverse practices of mandatory public annuity programs in various countries, such as the choice among several public annuity products, and the limited choice in the amount of public annuity purchase within some range (Lau and Zhang, 2023, Table 1). We emphasize that while our model is based on observed features of mandatory public annuity programs with partial waiver as much as possible, the major purpose of our analysis is to illustrate the key messages of this paper using a simple model.

3.2 Health and wealth characteristics of the pensioners

In the model used in this paper, the health and wealth levels of pensioners are heterogeneous. The joint probability density function of health and wealth of gender i is denoted by $g(\theta, w; i)$, where $i = f$ for female pensioners and $i = m$ for male pensioners.[13]

To capture the stylized facts about these two variables in many countries, we make three key assumptions. The first two assumptions are about gender gaps. In most countries, women have longer life expectancy but men have higher level of wealth on average. For gender difference in health, we assume that

$$E(\theta; f) = \int_{\underline{\theta}}^{\overline{\theta}} \theta g_\theta(\theta; f) d\theta > \int_{\underline{\theta}}^{\overline{\theta}} \theta g_\theta(\theta; m) d\theta = E(\theta; m), \tag{2}$$

where $g_\theta(\theta; i) = \int_{\underline{w}}^{\overline{w}} g(\theta, w; i) dw$ is the marginal probability density function of health of gender i. For gender difference in wealth, we assume that

$$G_w(w; f) - G_w(w; m) > 0, \ \forall w \in (\underline{w}, \overline{w}), \tag{3}$$

where $G_w(w; i) = \int_{\underline{w}}^{w} g_w(w; i) dw$ is the cumulative distribution function of

[13] We modify the standard notations for the various probability density functions by adding the gender after the semi-colon, as in $g(., .; i)$, because of the need to distinguish the distributions of male and female pensioners. By defining the probability density functions in this way, we can keep our notations simple by, for example, using a common symbol θ to refer to the survival probability of an annuitant of either gender and not using θ^i for gender i.

wealth of gender i, and $g_w(w;i) = \int_{\underline{\theta}}^{\overline{\theta}} g(\theta, w; i) d\theta$ is the marginal probability density function of wealth of gender i.

Condition (3) means that the wealth level of male pensioners first-order stochastically dominates that of the female pensioners.[14] It is well known that (3) implies that

$$E(w; f) = \int_{\underline{w}}^{\overline{w}} w g_w(w; f) dw < \int_{\underline{w}}^{\overline{w}} w g_w(w; m) dw = E(w; m), \qquad (4)$$

meaning that the average wealth of male pensioners is higher (see Appendix A). We specify the stronger assumption (3) because it eliminates possible irregularities if the weaker assumption (4) is used.[15] Assumption (3) is commonly used in many analytical studies (such as Hadar and Russell, 1969).

The gender gap in health (in terms of life expectancy) and that in wealth are well documented. According to United Nations (2019, Table 2), the remaining life expectancy at age 65 is 15.6 years for men and 18.3 years for women. On the other hand, women generally have less pension wealth or lifetime income than men. According to Wave 13 of the Health and Retirement Study (HRS) data in 2016, women contribute less to their retirement accounts than men.[16] Moreover, the first-order stochastic dominance assumption (3) is consistent with empirical evidence (see Figure 2). Another related evidence is that women have less lifetime income than men by around 10 percent to 30 percent, based on 83 developed and developing countries (Morton et al., 2014). Blau and Kahn (2017) summarize the literature of the gender wage gap in the United States (US) and predict that this gap will remain for some time.

[Insert Figure 2 here.]

[14] A distribution function $P(w)$ first-order stochastically dominates another distribution function $Q(w)$ if $P(w) \leq Q(w)$ for all w, with a strict inequality over some interval. We use the strong version of first-order stochastic dominance in (3) for the analysis in this paper.

[15] If we use the weaker assumption (4), Lemma 2 in Section 5 may not hold even though the mean wealth of male pensioners is higher than that of the females. Specifically, if the mean wealth of males is lower than that of the females for poor pensioners, but is substantially higher than that of the females for wealthy pensioners, then (4) may still hold but (3) does not hold. Assumption (3), which is empirically relevant and ensures that the *gender gap in wealth* holds for *all subgroups* of the pensioners, eliminates this irregularity.

[16] We focus on pension wealth in our analysis of the mandatory public annuity program. Since a pensioner's retirement wealth under this program is contributed by the pensioner's cumulative contributions throughout the working years, together with investment returns, we use the more easily available evidence of gender gap in annual contributions to the retirement accounts as a proxy for gender gap in pension wealth.

The third assumption is about the correlation of health and wealth of the pensioners. The conventional assumption of positive correlation of these two variables is supported by unreported analysis of the HRS data, which shows that the pensioners' annual contributions to their retirement accounts and their self-reported probabilities of living up to age 75 are positively correlated.[17] In this paper, we further assume that θ and w for each gender satisfy the *linear conditional mean* specification with a *positive slope*, as follows:

$$E(\theta|w;i) = \int_{\underline{\theta}}^{\overline{\theta}} \theta g_{\theta|w}(\theta|w;i) d\theta = a^i + b^i w, \tag{5}$$

such that

$$b^i > 0, \tag{6}$$

where $g_{\theta|w}(\theta|w;i)$ is the conditional probability density function of gender i's health level given that the wealth level is w. The main reason of assuming linear conditional mean in our analysis is to eliminate the irregularity that the positive correlation of health and pension wealth is contributed by the highly positive correlation for those pensioners with substantial wealth, while the two variables are uncorrelated or even negatively correlated for poor pensioners.[18] The linear conditional mean specification is not very restrictive, and it includes the commonly-used bivariate normal distribution, as well as bivariate uniform distribution, etc.[19]

The useful properties from the linear conditional mean specification (in, for example, Lemma 10.3 of Ghahramani, 2005) allow us to obtain the following lemma.[20] The proof is given in the Online Appendix.

Lemma 1. *When health and wealth of either gender satisfy the linear conditional mean specification (5) with positive correlation (6), the condi-*

[17] As the gender gap in wealth discussed in the previous paragraph, we use annual contribution to the retirement account in Wave 13 of the HRS data to proxy for pensioner's wealth. The health-wealth correlation coefficient based on this dataset is not very strong, even though it is still positive, perhaps because of the rather small sample (1,404 observations for men, and 1,447 observations for women). On the other hand, the correlation of life expectancy at 40 years of age and household income in the dataset used in Chetty et al. (2016), based on more than 1.4 billion person-year observations, is stronger.

[18] If this irregularity is present, Proposition 1 will not hold because adverse selection in that environment requires the presence of positive health-wealth correlation for pensioners with low levels of wealth. Essentially, the linear conditional mean specification makes sure that if health and wealth of a gender group are positively correlated, this *positive correlation* is found in *all subgroups* of the pensioners.

[19] The linear conditional mean specification has also been used in prediction (as in Spinnewijn, 2017, Section I).

[20] After presenting our main results, we will discuss in Section 5 the advantage of adopting the linear conditional mean specification (5).

tional mean $E(\theta|w;i)$ can be expressed as:

$$E(\theta|w;i) = E(\theta;i) + \rho^i \frac{\sigma_\theta^i}{\sigma_w^i}\left[w - E(w;i)\right] \tag{7}$$

with

$$\rho^i > 0, \tag{8}$$

where $E(\theta;i)$ and $E(w;i)$ are defined in (2) and (4), σ_θ^i is the standard deviation of gender i's health, σ_w^i is the standard deviation of gender i's wealth, and $\rho^i = \frac{cov(\theta,w;i)}{\sigma_\theta^i \sigma_w^i}$ is the correlation coefficient of health and wealth of gender i.

3.3 The severity of adverse selection

The public annuity payout level is important to the well-being of the pensioners as well as the budget balance of the public annuity provider.[21] There is no consensus regarding how the public annuity payout is determined, and the actual determination method may be different across countries. For the following analysis, we consider two possible ways that the public annuity provider sets the annuity payout level: (a) determined by the zero-profit condition,[22] or (b) set at the actuarially fair level. We consider case (a) in Sections 4 and 5 and consider case (b) in Section 6.

We first focus on the case with the zero-profit condition and gender-based pricing, and develop a measure of the severity of adverse selection. For each gender group, the revenue from the public annuity purchase (at Period 1) is given by

$$\int_{\underline{w}}^{\overline{w}} \int_{\underline{\theta}}^{\overline{\theta}} \alpha(w) g(\theta, w; i) d\theta dw = \int_{\underline{w}}^{\overline{w}} \alpha(w) g_w(w;i) dw, \tag{9}$$

where the equality in (9) holds because $\alpha(w)$ does not depend on the survival probability. On the other hand, the expected present discounted value of all annuity payments (measured in Period 1) by the public annuity provider is

$$\frac{1}{1+r} \int_{\underline{w}}^{\overline{w}} \int_{\underline{\theta}}^{\overline{\theta}} \theta A^i \alpha(w) g(\theta, w; i) d\theta dw, \tag{10}$$

[21] Since there is no annuitization choice under the mandatory public annuity program considered in this paper, we do not need to specify the pensioners' utility functions. Nevertheless, it is obvious that the annuity payout level, which is an element of a pensioner's budget constraint, affects the well-being of the pensioner. We follow this simple idea to develop a measure of the severity of adverse selection, and use this measure to obtain further results regarding the pensioners' welfare.

[22] The premium pension system in Sweden is a defined-contribution system with financial stability. It follows the practice that "every year net income is more or less equal to zero" (Swedish Pension Agency, 2020, p. 5), which is consistent with the zero-profit condition.

where A^i is the public annuity payout level and r is the risk-free interest rate from Period 1 to Period 2. When A^i is determined by the zero-profit condition (as in Abel, 1986; Villeneuve, 2003; Hosseini, 2015), we equate (9) and (10) to obtain

$$A^i = A^i_{zp} = \frac{(1+r)\int_{\underline{w}}^{\overline{w}} \alpha(w) g_w(w;i)dw}{\int_{\underline{w}}^{\overline{w}} \int_{\underline{\theta}}^{\overline{\theta}} \theta\alpha(w)g(\theta,w;i)d\theta dw}. \tag{11}$$

Under the zero-profit condition, a useful measure of the severity of adverse selection in the annuity market is[23]

$$\lambda^i \equiv \frac{1+r}{A^i_{zp}} - E(\theta;i) = \frac{\int_{\underline{w}}^{\overline{w}} \int_{\underline{\theta}}^{\overline{\theta}} \theta\alpha(w)g(\theta,w;i)d\theta dw}{\int_{\underline{w}}^{\overline{w}} \alpha(w) g_w(w;i)dw} - \int_{\underline{\theta}}^{\overline{\theta}} \theta g_\theta(\theta;i)d\theta. \tag{12}$$

According to (12), the severity of adverse selection is measured by the difference of the *annuitization-weighted average* of survival probability of group i and the *unweighted average* (i.e., expected value) of survival probability of that group.[24]

Based on the assumption that the public annuity payout is determined according the zero-profit condition, we will consider the implications of gender-based versus gender-neutral pricing on the severity of adverse selection in Sections 4 and 5, respectively.

4 Gender-based pricing in mandatory annuity programs: Adverse selection is the norm

We first consider the mandatory annuity program with partial waiver and under gender-based pricing. In this case, there is only one policy intervention:

[23] An alternative interpretation of (12) is the difference of the break-even annuity price under a mandatory annuity program with partial waiver and the (hypothetical) break-even annuity price without the partial waiver. We thank Casey Rothschild for suggesting this interpretation.

[24] The interpretation of annuitization-weighted average of survival probability can be seen when the first term on the RHS of (12) is written as $\int_{\underline{\theta}}^{\overline{\theta}} \theta \left[\frac{\int_{\underline{w}}^{\overline{w}} \alpha(w)g(\theta,w;i)dw}{\int_{\underline{w}}^{\overline{w}} \alpha(w)g_w(w;i)dw}\right] d\theta$. Similar expressions have appeared in earlier papers, such as (8) of Villeneuve (2003), (1) of Fang et al. (2008) and (22) of Lau and Zhang (2023). Moreover, it is straightforward to show that $\lambda^i = \frac{cov(\theta,\alpha(w);i)}{E(\alpha(w);i)}$, after using (A1) in Appendix B. When adverse selection is present, $cov\,(\theta, \alpha(w); i) > 0$, leading to $\lambda^i > 0$. In the context of voluntary public annuity plans, Lau and Zhang (2023) use this measure to investigate the guarantee element and non-escalating payments of these plans.

mandating the public annuity purchase by pensioners according to (1). The main result under gender-based pricing is given in the following proposition.

Proposition 1. *Consider an economy with positive health-wealth correlation given by (7) and (8). When gender-based pricing with zero-profit condition is adopted in a mandatory annuity program with partial waiver, adverse selection is not eliminated.*

The proof is given in Appendix B. The key points in the proof can be summarized in the following equation:

$$\lambda^i = \frac{cov\,(\theta, \alpha(w); i)}{E\,(\alpha(w); i)} = \rho^i \frac{\sigma_\theta^i}{\sigma_w^i} \frac{cov\,(w, \alpha(w); i)}{E\,(\alpha(w); i)}, \tag{13}$$

which is positive under (1) and (8). It can be seen from the first equality of (13) that λ^i is positive when survival probability and annuity purchase of gender i are positively correlated.

To see the intuition of (13), we consider two special cases regarding $cov\,(w, \alpha(w); i)$ and ρ^i, respectively. We compare (1) with the hypothetical situation with a strict mandatory public annuity program (in which there is no waiver for any pensioner), given by

$$\alpha(w) = M.$$

Under this strict mandatory program, $cov\,(w, \alpha(w); i) = cov\,(w, M; i) = 0$. Thus, $\lambda^i = 0$, and there is no adverse selection. We also compare (8) with the hypothetical situation that there is no correlation between health and wealth of a particular gender. In this case,

$$\rho^i = 0$$

holds, leading to $cov\,(\theta, \alpha(w); i) = 0$ even though $cov\,(w, \alpha(w); i) > 0$ under the mandatory annuity program with partial waiver and gender-based pricing. Essentially, when health and wealth are uncorrelated, the mandated level of annuity purchase, while being dependent on wealth level (w) under this mandatory annuity program, is not correlated to the risk type (θ).

The above two comparisons allow us to see the intuition of Proposition 1. The middle term of (13) indicates that the well-known source of adverse selection in the annuity market is the correlation of annuity purchase and survival probability: adverse selection arises when the high-risk group of pensioners (those with higher survival probabilities) buy larger levels of annuity. Panel A of Figure 1 helps us see the role of partial waiver: w and $\alpha(w)$ is perfectly correlated for $w < \frac{M}{\gamma}$, but is uncorrelated for buyers with pension wealth above $\frac{M}{\gamma}$ because all of them purchase the mandated level of annuity. The second equality of (13) links the covariance of θ and $\alpha(w)$ to that of w and $\alpha(w)$. It highlights the role of correlation of health and wealth. In particular, if there is no correlation between these two variables, then any possible covariance of w and $\alpha(w)$ would not be translated to the covariance of θ and $\alpha(w)$.

To summarize, it is the combined effect of the *partial waiver component* of the mandatory annuity program (which leads to $cov\,(w, \alpha(w); i) > 0$) and the *positive health-wealth correlation* ($\rho^i > 0$) that leads to a positive value of $cov\,(\theta, \alpha(w); i)$ in the mandatory annuity with partial waiver program.

A by-product of Proposition 1 is that mandatory annuity purchase may not necessarily eliminate adverse selection. The conventional wisdom is that allowing annuitization choices in the presence of asymmetric information leads to adverse selection, but prohibiting annuitization choices by mandating everyone to purchase annuity eliminates adverse selection (Einav and Finkelstein, 2011). In the policy setting of the mandatory annuity program with partial waiver, together with a positive health-wealth correlation (as documented in many countries), adverse selection is not eliminated according to Proposition 1. Since both of these ingredients correspond to the features that are empirically observed, Proposition 1 provides an important counter-example to the conventional wisdom regarding the ability of mandating annuity purchase to eliminate adverse selection.

The primary contribution of the analysis of this section is that we identify the main economic factors leading to *adverse selection* under gender-based pricing *even when choice is not allowed* in annuity purchases under a mandatory annuity program. The identified factors turn out to be one major component in understanding the severity of adverse selection when gender-neutral pricing is adopted, which will be considered in the next section.

5 Gender-neutral pricing in mandatory annuity programs: Advantageous selection is possible

Starting from 2012, gender-based pricing has been banned in all insurance industries in the EU. The major reason of banning gender-based pricing is that "gender equality is a fundamental right in the European Union and the Court of Justice made clear that this also applies to insurance pricing."[25]

[25] See the press release by the European Commission: https://ec.europa.eu/commission/presscorner/detail/en/IP_12_1430.

Besides the EU, several states in the US also forbid gender-based pricing in insurance products, such as car insurance, health insurance and so on.[26]

In this section, we analyze the impact of adopting gender-neutral pricing in the mandatory annuity program with partial waiver on the severity of adverse selection.

Define the annuity payout under gender-neutral pricing and zero-profit condition as A_{zp}. Under gender-neutral pricing, both male and female pensioners participate in the same public annuity program and pay the annuity provider in Period 1, and those who survive in Period 2 receive the annuity payment. When the annuity payout term is determined according to the zero-profit condition, it is given by

$$A_{zp} = \frac{(1+r)\left[s^f \int_{\underline{w}}^{\overline{w}} \int_{\underline{\theta}}^{\overline{\theta}} \alpha(w) g(\theta, w; f) d\theta dw + \left(1-s^f\right) \int_{\underline{w}}^{\overline{w}} \int_{\underline{\theta}}^{\overline{\theta}} \alpha(w) g(\theta, w; m) d\theta dw\right]}{s^f \int_{\underline{w}}^{\overline{w}} \int_{\underline{\theta}}^{\overline{\theta}} \theta\alpha(w) g(\theta, w; f) d\theta dw + \left(1-s^f\right) \int_{\underline{w}}^{\overline{w}} \int_{\underline{\theta}}^{\overline{\theta}} \theta\alpha(w) g(\theta, w; m) d\theta dw}, \tag{14}$$

where $s^f \in (0,1)$ is the share of female pensioners in the economy. The measure of the severity of adverse selection under gender-neutral pricing is

$$\lambda \equiv \frac{1+r}{A_{zp}} - \left[s^f E(\theta; f) + \left(1-s^f\right) E(\theta; m)\right]$$

$$= \frac{s^f E(\theta\alpha(w); f) + \left(1-s^f\right) E(\theta\alpha(w); m)}{s^f E(\alpha(w); f) + \left(1-s^f\right) E(\alpha(w); m)} - \left[s^f E(\theta; f) + \left(1-s^f\right) E(\theta; m)\right]. \tag{15}$$

Useful results regarding the severity of adverse selection (λ) of the mandatory annuity program under gender-neutral pricing are stated in the following two propositions. First, we present a useful decomposition formula for λ in Proposition 2, which holds whether the gender gap in health or wealth exists or not, and whether health and wealth of a particular gender group are correlated or not.[27] The share of public annuity purchase by female pensioners, defined as

$$\beta^f = \frac{s^f E(\alpha(w); f)}{s^f E(\alpha(w); f) + \left(1-s^f\right) E(\alpha(w); m)}, \tag{16}$$

[26] See, for example, the press release by the Consumer Federation of America in 2019: https://consumerfed.org/press_release/california-prohibits-auto-insurance-companies-from-considering-gender-when-setting-prices/.

[27] Formula (17) in Proposition 2, which is based on the decomposition of within-group and between-group effects, also does not depend on whether the pensioners' annuity purchases are mandated by the government or based on their decisions. (See, for example, Lau and Ying (2023).) More generally, this formula is applicable not only in an environment with two gender groups studied in this paper, but also in other situations involving the pooling of two distinct groups (such as two regions in a country).

is useful in the proof of Proposition 2, which is presented in Appendix C.

Proposition 2. *When gender-neutral pricing with zero-profit condition is adopted in a mandatory annuity program with partial waiver, the severity of adverse selection in the annuity market is given by*

$$\lambda = \lambda_{wg} + \lambda_{bg}, \tag{17}$$

where

$$\lambda_{wg} = \beta^f \frac{cov\left(\theta, \alpha(w); f\right)}{E(\alpha(w); f)} + \left(1 - \beta^f\right) \frac{cov\left(\theta, \alpha(w); m\right)}{E\left(\alpha(w); m\right)} = \beta^f \lambda^f + \left(1 - \beta^f\right) \lambda^m \tag{18}$$

and

$$\lambda_{bg} = \frac{s^f \left(1 - s^f\right) \left[E\left(\theta; f\right) - E\left(\theta; m\right)\right] \left[E(\alpha(w); f) - E\left(\alpha(w); m\right)\right]}{s^f E(\alpha(w); f) + \left(1 - s^f\right) E(\alpha(w); m)}. \tag{19}$$

Second, we obtain a sufficient condition for the presence of advantageous selection ($\lambda < 0$) according to the following proposition.

Proposition 3. *Consider an economy with gender gap in health given by (2), gender gap in wealth given by (3) and positive health-wealth correlation given by (7) and (8). When gender-neutral pricing with zero-profit condition is adopted in a mandatory annuity program with partial waiver,*

(a) λ_{wg} *defined in (18) is positive, and* λ_{bg} *defined in (19) is negative; and*

(b) a sufficient condition for advantageous selection to appear is

$$s^f \rho^f \sigma_\theta^f \sigma_w^f + \left(1 - s^f\right) \rho^m \sigma_\theta^m \sigma_w^m$$

$$< s^f \left(1 - s^f\right) \left[E\left(\theta; f\right) - E\left(\theta; m\right)\right] \int_{\underline{w}}^{\frac{M}{\gamma}} \left[G_w(w; f) - G_w(w; m)\right] dw. \tag{20}$$

The proof of Proposition 3 uses the following two lemmas. (Each of the two lemmas has its own interesting interpretation, which will be elaborated.)

Lemma 2. *If (1) and (3) hold, then*

$$E\left(\alpha(w); f\right) - E\left(\alpha(w); m\right) = -\gamma \int_{\underline{w}}^{\frac{M}{\gamma}} \left[G_w(w; f) - G_w(w; m)\right] dw < 0. \tag{21}$$

Lemma 3. *If (1), (7) and (8) hold, then*

$$\frac{cov\left(\theta, \alpha(w); i\right)}{E\left(\alpha(w); i\right)} < \frac{cov\left(\theta, \gamma w; i\right)}{E\left(\gamma w; i\right)}. \tag{22}$$

The proofs of Lemma 2, Lemma 3 and Proposition 3 are presented in Appendix D.

Proposition 3 shares some similarities, as well as differences, with that of Proposition 1. The similarities lie in the importance of the covariance between health and annuity purchase of each gender. Proposition 1 shows that these two variables are positively correlated under the mandatory annuity program with gender-based pricing. Similarly, in the mandatory annuity program with gender-neutral pricing, the covariance between these two variables for either group contributes to the severity of adverse selection. This feature is reflected by the weighted average term λ_{wg} in (18).

The new element in the measure of the severity of adverse selection under gender-neutral pricing comes from the between-group term λ_{bg} in (19). We show in (19) that λ_{bg} depends on the product of gender gap in health and gender gap in public annuity purchase. The gender gap in health, defined as $E(\theta; f) - E(\theta; m)$, is positive under assumption (2). This feature is standard and easy to understand. On the other hand, the gender gap in annuity purchase, defined as $E(\alpha(w); f) - E(\alpha(w); m)$, needs further investigation. As observed in the proof of Lemma 2, a negative value of $E(\alpha(w); f) - E(\alpha(w); m)$ arises from the interaction of the partial waiver component of the mandatory public annuity program and the difference in the wealth of male and female pensioners. Because of (1) and (3), the amount of annuity purchase waived for male pensioners is lower, leading to a higher amount of public annuity purchase. Combining the gender gap in public annuity purchase (due to the gender gap in pension wealth) with the gender gap in health, we conclude that λ_{bg} in (19) is negative. The negative product (of the two gender gaps) in the between-group term λ_{bg} affects the severity of adverse selection in a way opposite to the within-group positive correlation term λ_{wg} in (18).

There is an interesting comparison of these two terms based on within-group and between-group risk classification. In the presence of asymmetric information in health, the *low-risk individuals within each group* are the pensioners with low survival probability. Since the less healthy group of either gender *purchases less* public annuities because of the partial waiver and the health-wealth correlation, a *within-group positive correlation effect* arises according to Proposition 1. On the other hand, when we compare the group of male pensioners with the female counterpart, the low-risk group is the group of male pensioners (with a lower mean survival probability). Since the male group turns out to have a higher level of average wealth, their annuity purchase is higher because of the smaller amount of public annuity purchase which is waived under (1). Unlike the within-group outcome, we have the opposite between-group outcome that *low-risk group* of male pensioners *pur-*

chases more public annuities. The *between-group negative correlation effect* may lead to advantageous selection, if it is stronger than that of the within-group positive correlation effect.

Part (b) of Proposition 3 provides a sufficient condition for advantageous selection to arise. The proof makes use of Lemma 3, which provides an upper bound of the severity of adverse selection of the within-group correlation term. The right-hand side (RHS) term of (22) can be interpreted as the severity of adverse selection of a hypothetical mandatory annuity program in which the annuity purchases of all pensioners are proportional to their wealth levels. Relative to this hypothetical case, it can be seen that the pensioners' annuity purchase are the same in these two programs when their wealth are below the threshold level $\frac{M}{\gamma}$ of (1). On the other hand, the annuity purchase for pensioners above this threshold is constant (at M) under the mandatory annuity program with partial waiver, leading to a lower level of the severity of adverse selection according to Lemma 3. The sufficient condition (20), which is expressed in terms of exogenous parameters only, has useful implications that advantageous selection is more likely to arise when (a) gender gap in health, $E(\theta; f) - E(\theta; m)$, is relatively large, causing a strong between-group negative correlation effect; (b) gender gap in wealth for low-wealth pensioners, $\int_{\underline{w}}^{\frac{M}{\gamma}} [G_w(w; f) - G_w(w; m)]\, dw$, is relatively large, causing a strong between-group negative correlation effect; or (c) the correlation coefficient of health and wealth of either group (ρ^f, ρ^m) is relatively small, causing a weak within-group positive correlation effect.[28]

To conclude, we find in Proposition 3 that in a mandatory annuity program with partial waiver, banning gender-based pricing leads to a positive within-group effect and a negative between-group effect, both of which are induced by the partial waiver. In particular, advantageous selection is present if the between-group effect is strong enough.

It is observed in Appendix D that the linear conditional mean assumption (5) is used in the proof of Proposition 3(b) but not in that of Proposition 3(a). If we replace assumptions (5) and (6) with the alternative assumption that health and wealth are positively correlated for pensioners of each gender group with wealth levels below M, then we still obtain $\lambda_{wg} > 0$ and $\lambda_{bg} < 0$ according to Proposition 3(a). Thus, advantageous selection is present when $\lambda_{wg} < -\lambda_{bg}$. Our main result regarding the possibility of advantageous selection under gender-neutral pricing still holds with this alternative assumption

[28] Unreported computational analysis shows that advantageous selection is possible in mandatory annuity programs with gender-neutral pricing. The comparative static results regarding the gender gap in health and the correlation between health and wealth are also confirmed.

regarding the correlation of health and wealth. We use the linear conditional mean specification in this paper because it has been used in the literature (such as in Spinnewijn, 2017) and some commonly used distributions such as bivariate normal or bivariate uniform distributions satisfy this assumption. Moreover, it enables us to obtain the additional result of Proposition 3(b) regarding a sufficient condition of advantageous selection in terms of the underlying factors of the model.

Proposition 3 provides a new result about the effect of banning categorical discrimination (based on gender) on the severity of adverse selection of the annuity market, complementing the results in Hoy (1982), Crocker and Snow (1986), Finkelstein et al. (2009) and Aquilina et al. (2017). It is also related to the literature regarding the source of advantageous selection in insurance markets. Hemenway (1990) and de Meza and Webb (2001) argue that advantageous selection in insurance markets may arise in models with unobserved heterogeneity in risk tolerance, in addition to the risk type that is emphasized in models with a single dimension of private information. Consumers who are less risk-tolerant insure more but also are more safety-conscious to avoid accidents. Advantageous selection is possible when those who buy more insurance have less accidents on average than those who insure less. Fang et al. (2008, p. 311) further suggest that any private information could function as a source of advantageous selection if it is positively correlated with the amount of insurance purchase but negatively correlated with risk type. The additional dimension of unobserved heterogeneity does not have to be risk tolerance or other preference factor. In fact, their empirical analysis suggests that advantageous selection is present in the Medigap insurance market with the cognitive ability of the insureds being the extra dimension of heterogeneity in this market.

The above papers examine the possibility of advantageous selection in various insurance markets by relying on an extra dimension of heterogeneity (in risk preference or in cognitive ability) besides risk type (survival probability in the context of the annuity product). On the other hand, this paper focuses on government policies and gender differences (in health and wealth). We show that in a model of mandatory annuity program with partial waiver, advantageous selection is impossible when gender-based pricing is adopted but is possible when it is banned. Our paper provides an alternative channel for the presence of advantageous selection.

6 Another way to determine the annuity payout level

In the previous sections, we assume the zero-profit condition in determining the annuity payout level, which has also been used in Abel (1986), Villeneuve (2003) and Hosseini (2015). The assumption is usually adopted to analyze markets with free entry and exit. When it is applied to the public sector, it has the interpretation that the government takes the financially neutral position. While whether the government is willing to take this position is an empirical question, an advantage of this assumption is that it offers a consistent way to compare different policies, because the correlation of survival probability and annuity purchase is reflected in the equilibrium annuity payout, as in (12), under the zero-profit condition.

Nevertheless, a lot of information input is required for the public annuity payout to be determined according to the zero-profit condition. In the current model, the government needs to know the correlation of health and wealth to calculate the annuity payout under this assumption. If this task is too demanding, the government may need to use a simpler method to determine the public annuity payout level. We now consider an alternative assumption which is less demanding on the information required by the public annuity provider in setting the payout level. We assume that the annuity payout is set at the actuarially fair level corresponding to the targeted group of public annuity buyers (pensioners of one gender for gender-based pricing, or male and female pensioners together for gender-neutral pricing). In this case, the public annuity provider only needs to know the mean survival probabilities of the two groups.

First, consider gender-based pricing. In the mandatory annuity program with partial waiver, the annuity payout level A^i is assumed to be at the actuarially fair payout level

$$A^i = A^i_{af} = \frac{1+r}{E(\theta;i)}. \tag{23}$$

Together with (9) and (10), the budget balance of the annuity provider, in terms of the expected value, is given by

$$S^i_{af} = \int_{\underline{w}}^{\overline{w}} \alpha(w) g_w(w;i)dw - \frac{1}{E(\theta;i)} \int_{\underline{w}}^{\overline{w}} \int_{\underline{\theta}}^{\overline{\theta}} \theta\alpha(w)g(\theta,w;i)d\theta dw. \tag{24}$$

To see the relation of λ^i (if the annuity payout is determined according to the zero-profit condition) and S^i_{af} (if the payout is set at the actuarially fair level) under gender-based pricing, we use (12) and (24) to obtain[29]

$$S_{af}^{i}=E\left(\alpha(w);i\right)-\frac{E\left(\theta\alpha(w);i\right)}{E(\theta;i)}=\frac{E\left(\alpha(w);i\right)}{E(\theta;i)}\left[E(\theta;i)-\frac{E\left(\theta\alpha(w);i\right)}{E\left(\alpha(w);i\right)}\right]$$
$$=-\left[\frac{E\left(\alpha(w);i\right)}{E(\theta;i)}\right]\lambda^{i}. \tag{25}$$

Since S_{af}^{i} and λ^{i} are negatively related, the result in Proposition 1, which is based on λ^{i} when the annuity payout is determined according to the zero-profit condition, has a corresponding result in terms S_{af}^{i} under the actuarially fair payout. This is stated in the following corollary.

Corollary 4. *Consider an economy with positive health-wealth correlation given by (7) and (8). When gender-based pricing with the actuarially fair payout level is adopted in a mandatory annuity program with partial waiver, the budget of the annuity provider is in deficit.*

It is easy to see that both Proposition 1 and Corollary 4 are caused by the positive correlation of survival probability and public annuity purchase of either gender. Under gender-based pricing, the positive correlation of these two variables leads to a positive value of the severity of adverse selection (if the annuity payout is set according to the zero-profit condition) or a deficit in the public annuity provider's budget balance (if the annuity payout is set at the actuarially fair level).

Under gender-neutral pricing, there is also a close relation of the severity of adverse selection λ (if the annuity payout is determined according to the zero-profit condition) and the annuity provider's budget balance S_{af} (if the payout is set at the actuarially fair level). Since the analysis under gender-neutral pricing is similar to that under gender-based pricing, we only briefly summarize the results. In an economy under gender-neutral pricing, the annuity payout at the actuarially fair level is given by:

$$A_{af}=\frac{1+r}{s^{f}E(\theta);f)+\left(1-s^{f}\right)E(\theta;m)}. \tag{26}$$

Similar to the derivation of (25) under gender-based pricing, we can use the definition of S_{af} and (15) to obtain

$$S_{af}=s^{f}E(\alpha(w);f)+\left(1-s^{f}\right)E(\alpha(w);m)-\frac{s^{f}E(\theta\alpha(w);f)+\left(1-s^{f}\right)E(\theta\alpha(w);m)}{s^{f}E(\theta);f)+\left(1-s^{f}\right)E(\theta;m)}$$

[29] Note that we focus on λ^{i} (with the budget balance of the public annuity provider being always zero) if the annuity payout level is determined according to the zero-profit condition and on S_{af}^{i} (with $\frac{1+r}{A_{af}^{i}}-E(\theta;i)$ being always zero) if it is at the actuarially fair level. The same point also applies to gender-neutral pricing.

$$= -\left[\frac{s^f E(\alpha(w)); f) + \left(1-s^f\right) E(\alpha(w); m)}{s^f E(\theta); f) + (1-s^f)\, E(\theta; m)}\right]\lambda. \qquad (27)$$

The result in Proposition 3 in terms of λ also has a corresponding result in terms S_{af} under the actuarially fair payout level, as stated in the following corollary.

Corollary 5. *Consider an economy with gender gap in health given by (2), gender gap in wealth given by (3) and positive health-wealth correlation given by (7) and (8). When gender-neutral pricing with the actuarially fair payout level is adopted in a mandatory annuity program with partial waiver, the budget of the annuity provider is in surplus if (20) holds.*

We observe from the results in Sections 4 to 6 that the severity of adverse selection, measured by λ^i for gender-based pricing or λ for gender-neutral pricing, is useful whether the annuity payout is determined according to the zero-profit condition or the actuarially fair level. Take the example of gender-neutral pricing, the severity of adverse selection (λ) is negatively related to the annuity payout level received by the pensioners under the assumption of zero-profit condition, according to (15).[30] When λ is negative (i.e., advantageous selection is present), the annuity payout received by the pensioners is higher than the actuarially fair level. On the other hand, if the annuity payout is set at the actuarially fair level, λ is also useful because it is negatively related to the budget balance of the public annuity provider. A negative value of λ means that the public annuity provider has a budget surplus, according to (27). These results suggest that for either one of these two ways in setting the annuity payout level, a different value of λ potentially has efficiency implications to the pensioners, especially if it is accompanied by appropriate transfer policies.[31]

[30] Since the annuity payout level is always positive but λ may be positive or negative, a more careful interpretation of this statement is as follows. If $\frac{1+r}{A_{zp}}$ is larger than $s^f E(\theta); f) + \left(1-s^f\right) E(\theta; m)$, then λ is positive according to (15). It is easy to see that both λ and A_{zp} are positive and they are negatively related. On the other hand, λ is negative if $\frac{1+r}{A_{zp}}$ is smaller than $s^f E(\theta); f) + \left(1-s^f\right) E(\theta; m)$. In this case, a more negative value of λ is associated with a higher value of A_{zp}.

[31] More generally, while the efficiency implications of adverse or advantageous selection is an important research topic, the issues involved are rather complicated and a consensus has not yet been reached. As an example, a standard view is that adverse selection leads to the crowding out of socially beneficially trade. Interestingly, a recent article by de Meza et al. (2021) suggests that adverse selection may also crowd in dysfunctional trades in which social cost exceeding social benefit. Moreover, Fang et al. (2008) point out that while the effect of the additional dimension of heterogeneity in multidimensional private information models may "cancel out the positive correlation between ex post risk and insurance coverage that arises in classic adverse selection models such as Rothschild and

7 Concluding remarks

It is generally believed that adverse selection is present in the annuity market because of asymmetric information. A standard solution to alleviate the associated inefficiency is to disallow selection by making the buyers' annuity purchases mandatory.

This paper revisits this conventional wisdom. We observe that in some countries (such as Denmark and Lithuania) adopting mandatory public annuity programs, there is a partial waiver in which the pensioners with low level of pension wealth are required to purchase less than the mandated level applicable to other pensioners (with wealth above the threshold level). This exemption clause makes good sense because requiring pensioners with lower level of wealth to buy the mandated annuity level may not be beneficial to them, particularly when the mandated level is reasonably high. Even though this waiver is not contingent on the variable (survival probability) directly relevant for risk classification, Proposition 1 shows that when health and pension wealth are correlated, adverse selection is not eliminated.

Another result is present when gender-based pricing is banned in this mandatory public annuity program. Banning gender-based pricing has been observed in many countries such as those in the EU after 2012. By decomposing the severity of adverse selection in this environment into the within-group and between-group components, we find that the within-group component exerts an upward pressure on the severity of adverse selection, while the between-group component has the opposite effect. Comparing Proposition 1 with Proposition 3, it can be seen that in a mandatory public annuity program with partial waiver, adverse selection is not eliminated under gender-based pricing but may be eliminated under gender-neutral pricing. In fact, banning gender-based pricing may even lead to the beneficial outcome of advantageous selection in which the pensioners receive the annuity payout at a level higher than the actuarially fair level if the between-group effect is strong enough. Using models with multidimensional private information, Hemenway (1990), de Meza and Webb (2001) and Fang et al. (2008) suggest that advantageous selection may arise when there is an extra dimension of heterogeneity, in addition to the risk type commonly assumed. Our paper, on the other hand, suggests that even in a model with one source of private information, advantageous selection may still arise from the interaction of

Stiglitz (1976), it is important to emphasize that this does not mean there is no inefficiency in such a market" (p. 343). Investigating how gender-based or gender-neutral pricing in various settings with adverse or advantageous selection may affect different aspects of efficiency is an interesting topic, but it would digress too much from the main message of this paper. It will be left to future research.

two policy interventions: the partial waiver element in the mandatory public annuity program and the use of gender-neutral pricing.

We prove the above results under the assumption of zero-profit condition. This assumption leads to a nice property that the correlation of survival probability and annuity purchase is reflected in the equilibrium annuity payout value, under either gender-based or gender-neutral pricing. However, calculating the equilibrium annuity payout based on the zero-profit assumption requires a full set of information including the distribution of survival probability for each gender group, which can be quite demanding. In practice, the government may simply adopt the actuarially fair criterion out of convenience, because the actuarially fair payout level requires only the information about the average survival probability of the pensioners. When the actuarially fair criterion is adopted, we show that the correlation of survival probability and annuity purchase affects the annuity provider's budget balance. In particular, the mandatory annuity program with gender-based pricing leads to a budget deficit, because the program overpays the annuitants, compared with the situation under the zero-profit condition. On the other hand, the mandatory annuity program with gender-neutral pricing underpays the annuitants when condition (20) holds, thereby leading to a budget surplus. These results are useful to annuity providers adopting the actuarially fair criterion.

The two main results (Propositions 1 and 3) are most clearly demonstrated in a mandatory public annuity program with partial waiver. While this annuity program is adopted by some countries, one may wonder whether similar results are also present in a different program in which the pensioners' annuity purchase are based on their choices instead.[32] In analyzing the impact of introducing deferred annuities under gender-neutral pricing, Lau and Ying (2023) consider a model of deferred and immediate annuity purchase choices. Using the decomposition formula in (17) of Proposition 2, it is found that the within-group correlation effect in the deferred annuity market is zero. As a result, advantageous selection in deferred annuities is present if the between-group correlation effect is negative, which happens when the effect of survival probability on deferred annuity purchase (leading to higher purchase by women) is smaller than the effect of wealth on such purchase (leading to higher purchase by men). When gender-neutral pricing

[32]When the pensioners are allowed to have choices (even limited ones) in a mandatory annuity program, their decisions may lead to partial offsetting effect to the mandatory policy. Finkelstein et al. (2009) study the UK annuity market in which participation is mandatory but the pensioners have choices among a range of annuity products with different characteristics. In this environment that endogenous adjustment is possible, they find evidence of welfare loss of the low-risk group under gender-neutral pricing, but the loss is not substantial.

is imposed, it is quite possible for advantageous selection to arise either under a mandatory immediate annuity program with partial waiver (as shown in this paper) or a deferred annuity program in which the pensioners make annuitization choices (as shown in Lau and Ying, 2023).

8 Appendix

In Appendix A, we present empirical evidence which supports the assumption of gender gap in wealth. We then prove Propositions 1 and 2 in Appendices B and C, respectively. Finally, we present the proofs of Lemma 2, Lemma 3 and Proposition 3 in Appendix D.

8.1 Appendix A: The gender gap in wealth

We use the annual contributions to pensioners' defined-contribution plans from the Health and Retirement Study (HRS) as a proxy for their lifetime retirement wealth. The HRS is a national panel survey of individuals over age 50 and their spouses. We use Wave 13 of HRS that was conducted in 2016.

We use the data to generate the cumulative distribution functions of annual contributions to defined-contribution plans by male and female pensioners in Figure 2. The two distribution functions are consistent with (3). The assumption in (3) leads to a useful property, according to the following lemma.

Lemma A1. *If a distribution $P(w)$ first-order stochastically dominates another distribution $Q(w)$, then the mean of distribution $P(w)$ is higher than the mean of distribution $Q(w)$.*

Lemma A1 is a well-known result. (See, for example, Hadar and Russell (1969), Theorem 1'.)

Using Lemma A1, (3) implies that (4) holds. According to the HRS data in 2016, the average annual contributions to defined-contribution plans of males ($6,850) is around 45% higher than that for females ($4,742).[33] Some summary statistics about this variable of the two groups are presented in Table A1. The evidence is consistent with (4).

[Insert Table A1 here.]

[33] We eliminate the outliers of this variable by excluding the contributions below the bottom 1% or above the top 1%.

8.2 Appendix B: Proof of Proposition 1

First, we use (11), (12) and

$$E(\theta\alpha(w);i)=cov(\theta,\alpha(w);i)+E(\theta;i)E\left(\alpha(w);i\right) \tag{A1}$$

to obtain

$$\begin{aligned}\lambda^i &= \frac{E\left(\theta\alpha(w);i\right)}{E\left(\alpha(w);i\right)}-E(\theta;i)\\ &= \frac{cov\left(\theta,\alpha(w);i\right)+E\left(\theta;i\right)E\left(\alpha(w);i\right)}{E\left(\alpha(w);i\right)}-E(\theta;i),\end{aligned}$$

which leads to the first equality of (13).

Second,

$$\begin{aligned}cov\left(\theta,\alpha(w);i\right) &= \int_{\underline{w}}^{\overline{w}}\int_{\underline{\theta}}^{\overline{\theta}}\theta\alpha(w)g(\theta,w;i)d\theta dw-E\left(\theta;i\right)E\left(\alpha(w);i\right)\\ &= \int_{\underline{w}}^{\overline{w}}\alpha(w)g_w(w;i)\left[\int_{\underline{\theta}}^{\overline{\theta}}\theta g_{\theta|w}(\theta|w;i)d\theta\right]dw-E\left(\theta;i\right)E\left(\alpha(w);i\right)\\ &= \int_{\underline{w}}^{\overline{w}}\alpha(w)g_w(w;i)\left\{E(\theta;i)+\rho^i\frac{\sigma_\theta^i}{\sigma_w^i}\left[w-E(w;i)\right]\right\}dw-E\left(\theta;i\right)E\left(\alpha(w);i\right)\\ &= \rho^i\frac{\sigma_\theta^i}{\sigma_w^i}\int_{\underline{w}}^{\overline{w}}\alpha(w)\left[w-E(w;i)\right]g_w(w;i)dw\\ &= \rho^i\frac{\sigma_\theta^i}{\sigma_w^i}cov\left(w,\alpha(w);i\right),\end{aligned} \tag{A2}$$

where we have used $g_{\theta|w}(\theta|w;i)=\frac{g(\theta,w;i)}{g_w(w;i)}$ in the second line and (7) in the third line. Using (A2), we obtain the second equality of (13).

Finally, define a Chebyshev's Sum function

$$C^i=\int_{\underline{w}}^{\overline{w}}\int_{\underline{w}}^{\overline{w}}(x-y)\left[\alpha(x)-\alpha(y)\right]g_w(x;i)g_w(y;i)dxdy, \tag{A3}$$

where x and y are two arbitrary indexes. According to (1), $\alpha(w)$ is a weakly increasing function with a strictly increasing interval. Therefore,

$$C^i>0. \tag{A4}$$

Moreover, by expanding the RHS of (A3) and simplifying, we obtain

$$C^i=2\int_{\underline{w}}^{\overline{w}}\int_{\underline{w}}^{\overline{w}}x\alpha(x)g_w(x;i)g_w(y;i)dxdy-2\int_{\underline{w}}^{\overline{w}}\int_{\underline{w}}^{\overline{w}}x\alpha(y)g_w(x;i)g_w(y;i)dxdy$$

$$= 2\left[\int_{\underline{w}}^{\overline{w}} x\alpha(x)g_w(x;i)dx\right]\left[\int_{\underline{w}}^{\overline{w}} g_w(y;i)dy\right] - 2\left[\int_{\underline{w}}^{\overline{w}} xg_w(x;i)dx\right]\left[\int_{\underline{w}}^{\overline{w}} \alpha(y)g_w(y;i)dy\right]$$

$$= 2E\left(w\alpha(w);i\right) - 2E(w;i)E\left(\alpha(w);i\right)$$

$$= 2cov\left(w,\alpha(w);i\right), \tag{A5}$$

where we have used $\int_{\underline{w}}^{\overline{w}} g_w(y;i)dy = 1$ and replaced the arbitrary indexes x and y by w in the third line.

Combining (A2), (A4) and (A5), we conclude that $cov\left(\theta,\alpha(w);i\right) > 0$. This proves Proposition 1. ■

8.3 Appendix C: Proof of Proposition 2

Together with (A1), we can express (15) as

$$\lambda = \frac{s^f cov\left(\theta,\alpha(w);f\right) + \left(1-s^f\right) cov\left(\theta,\alpha(w);m\right)}{s^f E(\alpha(w);f) + \left(1-s^f\right) E(\alpha(w);m)}$$

$$+\frac{s^f E(\theta;f)E(\alpha(w);f) + \left(1-s^f\right) E(\theta;m)E(\alpha(w);m)}{s^f E(\alpha(w);f) + \left(1-s^f\right) E(\alpha(w);m)}$$

$$-\left[s^f E\left(\theta;f\right) + \left(1-s^f\right) E\left(\theta;m\right)\right]. \tag{A6}$$

Using (13) and (16), it can be shown that the first term on the RHS of (A6) is λ_{wg} in (18). Based on straightforward but tedious algebra, it can be shown that the sum of the second and third terms on the RHS of (A6) is λ_{bg} in (19). ■

8.4 Appendix D: Proof of Proposition 3 and related proofs

We first present the proofs of Lemma 2 and Lemma 3. Based on these results, we then prove Proposition 3.

8.4.1 Proof of Lemma 2

Based on (1), the mean annuity purchase of the two groups differ by

$$E\left(\alpha(w);f\right) - E\left(\alpha(w);m\right)$$

$$= \left[\int_{\underline{w}}^{\frac{M}{\gamma}} \gamma w g_w(w;f)dw + \int_{\frac{M}{\gamma}}^{\overline{w}} M g_w(w;f)dw\right] - \left[\int_{\underline{w}}^{\frac{M}{\gamma}} \gamma w g_w(w;m)dw + \int_{\frac{M}{\gamma}}^{\overline{w}} M g_w(w;m)dw\right]$$

$$= M \int_{\underline{w}}^{\overline{w}} g_w(w; f)dw + \int_{\underline{w}}^{\frac{M}{\gamma}} (\gamma w - M)\, g_w(w; f)dw$$

$$-M \int_{\underline{w}}^{\overline{w}} g_w(w; m)dw - \int_{\underline{w}}^{\frac{M}{\gamma}} (\gamma w - M)\, g_w(w; m)dw$$

$$= \int_{\underline{w}}^{\frac{M}{\gamma}} (\gamma w - M) \left[g_w(w; f) - g_w(w; m)\right] dw. \tag{A7}$$

Applying integration by parts to the RHS term of (A7), we have

$$\int_{\underline{w}}^{\frac{M}{\gamma}} (\gamma w - M) \left[g_w(w; f) - g_w(w; m)\right] dw$$

$$= (\gamma w - M) \left[G_w(w; f) - G_w(w; m)\right] \Big|_{\underline{w}}^{\frac{M}{\gamma}} - \int_{\underline{w}}^{\frac{M}{\gamma}} \gamma \left[G_w(w; f) - G_w(w; m)\right] dw$$

$$= -\gamma \int_{\underline{w}}^{\frac{M}{\gamma}} \left[G_w(w; f) - G_w(w; m)\right] dw, \tag{A8}$$

where the first term in the second line is zero, because $G_w(\underline{w}; f) - G_w(\underline{w}; m) = 0$ for the lower limit and $\gamma \left(\frac{M}{\gamma}\right) - M = 0$ for the upper limit.

Combining (3), (A7) and (A8), we obtain Lemma 2. ■

8.4.2 Proof of Lemma 3

Similar to (A2), we have

$$cov(\theta, \gamma w; i) = \rho^i \frac{\sigma_\theta^i}{\sigma_w^i} \int_{\underline{w}}^{\overline{w}} \gamma w \left[w - E(w; i)\right] g_w(w; i)dw. \tag{A9}$$

Combining (A2) and (A9), we obtain

$$\frac{cov\,(\theta, \alpha(w); i)}{E\,(\alpha(w); i)} - \frac{cov\,(\theta, \gamma w; i)}{E\,(\gamma w; i)}$$

$$= \rho^i \frac{\sigma_\theta^i}{\sigma_w^i} \left[\frac{\int_{\underline{w}}^{\overline{w}} \alpha(w) \left[w - E(w; i)\right] g_w(w; i)dw}{E\,(\alpha(w); i)} - \frac{\int_{\underline{w}}^{\overline{w}} \gamma w \left[w - E(w; i)\right] g_w(w; i)dw}{E\,(\gamma w; i)}\right]$$

$$= \rho^i \frac{\sigma_\theta^i}{\sigma_w^i} \left[\int_{\underline{w}}^{\overline{w}} wp\,(w; i)\, dw - \int_{\underline{w}}^{\overline{w}} wq\,(w; i)\, dw\right] \tag{A10}$$

after cancellation of similar terms, where

$$p(w;i) = \frac{\alpha(w)}{E(\alpha(w);i)} g_w(w;i) \tag{A11}$$

and

$$q(w;i) = \frac{\gamma w}{E(\gamma w;i)} g_w(w;i). \tag{A12}$$

It can be seen from (A11) and (A12) that

$$\int_{\underline{w}}^{\overline{w}} p(w;i)\,dw = \int_{\underline{w}}^{\overline{w}} q(w;i)\,dw = 1. \tag{A13}$$

Therefore, $p(w;i)$ and $q(w;i)$ can be interpreted as probability density functions, induced by $\alpha(w)$ and γw, respectively. We obtain from (1) that

$$E(\alpha(w);i) < E(\gamma w;i), \tag{A14}$$

which is useful when comparing (A11) with (A12).

When $w \in [\underline{w}, \frac{M}{\gamma})$, we know from (1) that $\alpha(w) = \gamma w$. Together with (A14), we have $\frac{1}{g_w(w;i)}[p(w;i) - q(w;i)] = \frac{\gamma w}{E(\alpha(w);i)} - \frac{\gamma w}{E(\gamma w;i)} > 0$.

Next, consider $w \in \left[\frac{M}{\gamma}, \overline{w}\right]$. We obtain three important results. First, $\frac{p(w;i)}{g_w(w;i)} = \frac{\alpha(w)}{E(\alpha(w);i)}$ is constant but $\frac{q(w;i)}{g_w(w;i)} = \frac{\gamma w}{E(\gamma w;i)}$ is increasing in w. Second, when $w = \frac{M}{\gamma}$, the beginning of this interval, we have $\frac{1}{g_w(\frac{M}{\gamma};i)}\left[p\left(\frac{M}{\gamma};i\right) - q\left(\frac{M}{\gamma};i\right)\right] = \frac{M}{E(\alpha(w);i)} - \frac{M}{E(\gamma w;i)} > 0$. Third, when $w = \overline{w}$, the end of this interval, we have

$$\frac{1}{g_w(\overline{w};i)}[p(\overline{w};i) - q(\overline{w};i)] = \frac{M}{E(\alpha(w);i)} - \frac{\overline{w}}{E(w;i)}$$

$$= \frac{M\left[\int_{\underline{w}}^{\frac{M}{\gamma}} w g_w(w;i)dw + \int_{\frac{M}{\gamma}}^{\overline{w}} w g_w(w;i)dw\right] - \overline{w}\left[\int_{\underline{w}}^{\frac{M}{\gamma}} \gamma w g_w(w;i)dw + \int_{\frac{M}{\gamma}}^{\overline{w}} M g_w(w;i)dw\right]}{E(\alpha(w);i)\,E(w;i)}$$

$$= \frac{(M - \gamma\overline{w})\int_{\underline{w}}^{\frac{M}{\gamma}} w g_w(w;i)dw + M\int_{\frac{M}{\gamma}}^{\overline{w}} (w - \overline{w}) g_w(w;i)dw}{E(\alpha(w);i)\,E(w;i)} < 0,$$

where the inequality arises because $M - \gamma\overline{w} < 0$ and $w - \overline{w} < 0$.

Combining the above results, we conclude that $p(w;i)$ and $q(w;i)$ cross once, and it happens between $\frac{M}{\gamma}$ and $\overline{w}$. (See Panel B of Figure 1.)

Based on the single-crossing feature, define the critical value w_c^i such that

$$p(w;i)\begin{cases} > q(w;i), & w < w_c^i \\ = q(w;i), & w = w_c^i \\ < q(w;i), & w > w_c^i \end{cases}. \tag{A15}$$

Define $P(w;i)$ (resp. $Q(w;i)$) as the cumulative distribution function corresponding to $p(w;i)$ (resp. $q(w;i)$). When $w \in [\underline{w}, w_c^i]$, we use (A15) to obtain

$$P(w;i) - Q(w;i) = \int_{\underline{w}}^{w} p(\xi;i)d\xi - \int_{\underline{w}}^{w} q(\xi;i)d\xi = \int_{\underline{w}}^{w} [p(\xi;i) - q(\xi;i)]\, d\xi > 0, \tag{A16}$$

where ξ is an index of pension wealth. When $w \in (w_c^i, \overline{w}]$, we use (A13) and (A15) to obtain

$$P(w;i) - Q(w;i) = \int_{\underline{w}}^{w} [p(\xi;i) - q(\xi;i)]\, d\xi$$

$$= \int_{\underline{w}}^{\overline{w}} [p(\xi;i) - q(\xi;i)]\, d\xi - \int_{w}^{\overline{w}} [p(\xi;i) - q(\xi;i)]\, d\xi$$

$$= -\int_{w}^{\overline{w}} [p(\xi;i) - q(\xi;i)]\, d\xi > 0. \tag{A17}$$

Combining (A16) and (A17), we conclude that the distribution $Q(w;i)$ first-order stochastically dominates $P(w;i)$. Using Lemma A1 in Appendix A (which is based on Theorem 1' in Hadar and Russell, 1969), we conclude that the mean of $Q(w;i)$, given by $\int_{\underline{w}}^{\overline{w}} wq(w;i)\, dw$, is higher than the mean of $P(w;i)$, given by $\int_{\underline{w}}^{\overline{w}} wp(w;i)\, dw$. Together with (A10), we prove Lemma 3. ■

8.4.3 Proof of Proposition 3

The term λ_{wg} in (18) is positive, based on (1) and (8). The term λ_{bg} in (19) is negative, because of Lemma 2. This proves part (a).

We obtain an upper bound of λ_{wg} as follows. Based on Lemma 1, we have

$$cov(\theta, \gamma w; i) = \rho^i \frac{\sigma_\theta^i}{\sigma_w^i} cov(w, \gamma w; i) = \gamma \rho^i \sigma_\theta^i \sigma_w^i. \tag{A18}$$

Using (16) and (18), we obtain

$$\left[s^f E(\alpha(w);f) + \left(1 - s^f\right) E(\alpha(w);m)\right] \lambda_{wg}$$

$$= s^f E(\alpha(w);f) \frac{cov(\theta, \alpha(w);f)}{E(\alpha(w);f)} + \left(1 - s^f\right) E(\alpha(w);m) \frac{cov(\theta, \alpha(w);m)}{E(\alpha(w);m)}$$

$$< s^f E(\alpha(w);f) \frac{cov(\theta, \gamma w;f)}{E(\gamma w;f)} + \left(1 - s^f\right) E(\alpha(w);m) \frac{cov(\theta, \gamma w;m)}{E(\gamma w;m)}$$

$$
\begin{aligned}
&< s^f E(\gamma w; f) \frac{\gamma \rho^f \sigma_\theta^f \sigma_w^f}{E(\gamma w; f)} + \left(1 - s^f\right) E(\gamma w; m) \frac{\gamma \rho^m \sigma_\theta^m \sigma_w^m}{E(\gamma w; m)} \\
&= \gamma \left[s^f \rho^f \sigma_\theta^f \sigma_w^f + \left(1 - s^f\right) \rho^m \sigma_\theta^m \sigma_w^m \right],
\end{aligned}
$$

where Lemma 3 has been used in the first inequality, and (A14) and (A18) have been used in the second inequality.

Therefore, a sufficient condition for $\lambda_{wg} < -\lambda_{bg}$ is

$$
\frac{\gamma \left[s^f \rho^f \sigma_\theta^f \sigma_w^f + \left(1 - s^f\right) \rho^m \sigma_\theta^m \sigma_w^m \right]}{s^f E(\alpha(w); f) + (1 - s^f) E(\alpha(w); m)} < -\lambda_{bg},
$$

which leads to (20) after using Lemma 2. This proves part (b). ■

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References

[1] Abel, A. B. (1986), Capital accumulation and uncertain lifetimes with adverse selection. Econometrica 54(5), 1079-1097.

[2] Akerlof, G. A. (1970), The market for “lemons”: Quality uncertainty and the market mechanism. Quarterly Journal of Economics 84(3), 488-500.

[3] Aquilina, M., Baker, R., Majer, T. (2017), The money’s worth of annuities in the UK between 2006 and 2014. Journal of the Economics of Ageing 9, 208-218.

[4] Benartzi, S., Previtero, A., Thaler, R. H. (2011), Annuitization puzzles. Journal of Economic Perspectives 25(4), 143-164.

[5] Blau, F. D., Kahn, L. M. (2017), The gender wage gap: Extent, trends, and explanations. Journal of Economic Literature 55(3), 789-865.

[6] Cawley, J., Philipson, T. (1999), An empirical examination of information barriers to trade in insurance. American Economic Review, 89(4), 827-846.

[7] Chetty, R., Stepner, M., Abraham, S., Lin, S., Scuderi, B., Turner, N., Bergeron, A., Cutler, D. (2016), The association between income and life expectancy in the United States, 2001-2014. Journal of American Medical Association 315(16), 1750-1766.

[8] Crocker, K. J., Snow, A. (1986), The efficiency effects of categorical discrimination in the insurance industry. Journal of Political Economy 94(2), 321-344.

[9] de Meza, D., Reito, F., Reyniers (2021), Too much trade: The hidden problem of adverse selection. Journal of Public Economics 204, 104551.

[10] de Meza, D., Webb, D. C. (2001), Advantageous selection in insurance markets. RAND Journal of Economics, 32(2), 249-262.

[11] Diamond, P. (2004), Social security. American Economic Review 94(1), 1-24.

[12] Einav, L., Finkelstein, A. (2011), Selection in insurance markets: Theory and empirics in pictures. Journal of Economic Perspectives 25(1), 115-38.

[13] Fang, H., Keane, M. P., Silverman, D. (2008), Sources of advantageous selection: Evidence from the Medigap insurance market. Journal of political Economy 116(2), 303-350.

[14] Finkelstein, A., McGarry, K. (2006), Multiple dimensions of private information: evidence from the long-term care insurance market. American Economic Review 96(4), 938-958.

[15] Finkelstein, A., Poterba, J. (2002), Selection effects in the United Kingdom individual annuities market. Economic Journal 112(476), 28-50.

[16] Finkelstein, A., Poterba, J. (2004), Adverse selection in insurance markets: Policyholder evidence from the UK annuity market. Journal of Political Economy 112(1), 183-208.

[17] Finkelstein, A., Poterba, J., Rothschild, C. (2009), Redistribution by insurance market regulation: Analyzing a ban on gender-based retirement annuities. Journal of Financial Economics, 91(1), 38-58.

[18] Friedman, B. M., Warshawsky, M. J. (1990), The cost of annuities: Implications for saving behavior and bequests. Quarterly Journal of Economics 105(1), 135-154.

[19] Ghahramani, S. (2005), Fundamentals of probability with stochastic processes (3rd edition). N.J.: Pearson Prentice Hall.

[20] Hadar, J., Russell, W. R. (1969), Rules for ordering uncertain prospects. American Economic Review 59(1), 25-34.

[21] Hemenway, D. (1990), Propitious selection. Quarterly Journal of Economics 105(4), 1063-1069.

[22] Hosseini, R. (2015), Adverse selection in the annuity market and the role for social security. Journal of Political Economy 123, 941-984.

[23] Hoy, M. (1982), Categorizing risks in the insurance industry. The Quarterly Journal of Economics 97(2), 321-336.

[24] Lau, S.-H. P., Ying, Y. (2023), Deferred annuities with gender-neutral pricing: Benefitting most women without adversely affecting too many men. Working paper, University of Hong Kong.

[25] Lau, S.-H. P., Zhang, Q. (2023), A common thread linking the design of guarantee and nonescalating payments of public annuities. Journal of Risk and Insurance 90(3), 703-742.

[26] Mandatory Provident Fund Schemes Authority (2015), Towards Retirement Security. Hong Kong: Mandatory Provident Fund Schemes Authority. (https://www.mpfa.org.hk/eng/anni_publication/towards_retirement_security.pdf)

[27] Morton, M., Klugman, J., Hanmer, L., Singer, D. (2014), Gender at Work: A Companion to the World Development Report on Jobs. Washington, DC: World Bank.

[28] OECD (2019), Health at a Glance 2019: OECD Indicators. OECD Publishing, Paris. (https://www.oecd-ilibrary.org/social-issues-migration-health/health-at-a-glance-2019_4dd50c09-en)

[29] OECD (2021), Pensions at a Glance 2021: OECD and G20 Indicators. OECD Publishing, Paris. (https://doi.org/10.1787/ca401ebd-en)

[30] Rothschild, M., Stiglitz, J. (1976), Equilibrium in competitive insurance markets: An essay on the economics of imperfect information. Quarterly Journal of Economics 90(4), 629-649.

[31] Spinnewijn, J. (2017), Heterogeneity, demand for insurance, and adverse selection. American Economic Journal: Economic Policy 9(1), 308-343.

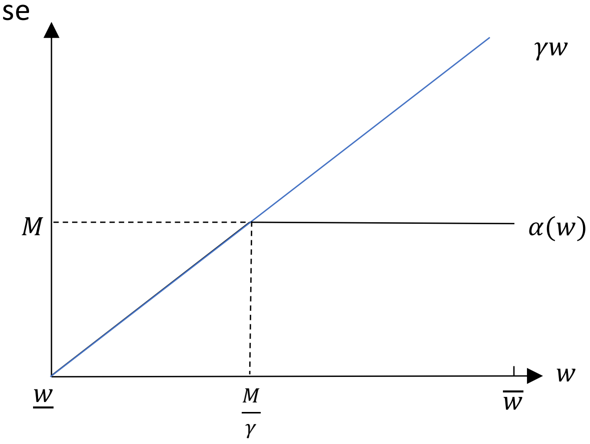
[32] Swedish Pension Agency (2020), Orange Report 2020: Annual Report of the Swedish Pension System. Sweden: Swedish Pension Agency.

[33] United Nations (2019), World Population Ageing 2019: Highlights. Department of Economic and Social Affairs, Population Division.

[34] Villeneuve, B. (2003), Mandatory pensions and the intensity of adverse selection in life insurance markets. Journal of Risk and Insurance 70(3), 527-548.

[35] Yaari, M. E. (1965), Uncertain lifetime, life insurance, and the theory of the consumer. Review of Economic Studies 32(2), 137-150.

Panel A: Mandated annuity purchase (for either gender group)



Panel B: Single-crossing feature

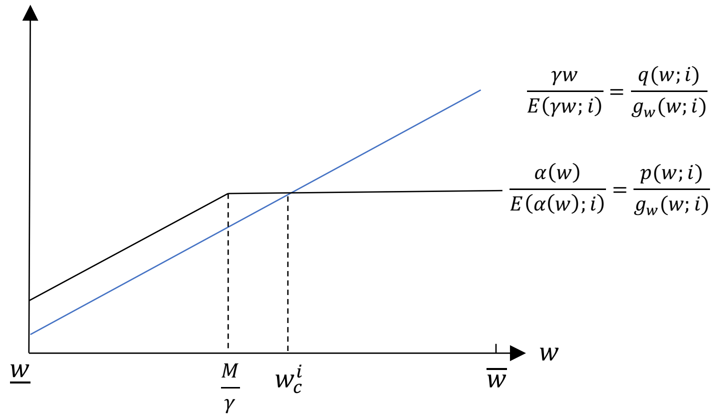


Figure 1: Mandatory annuity program with partial waiver

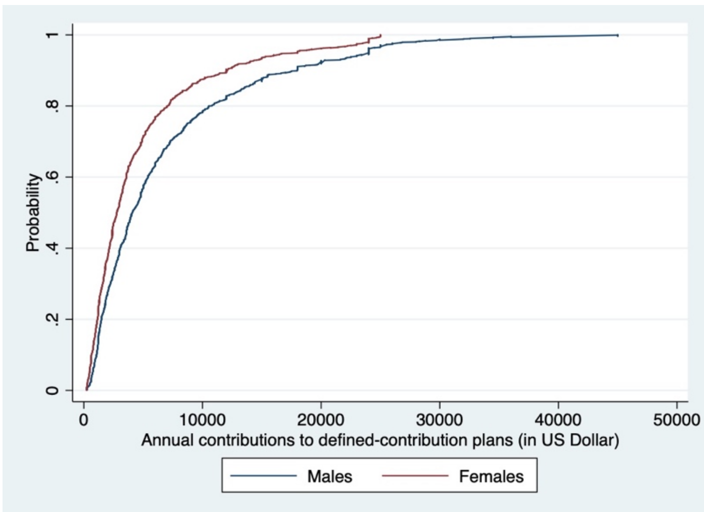


Figure 2: Cumulative distribution functions of annual contributions to defined-contribution plans by US male and female pensioners

	Observations	Mean ($)	Min ($)	Max ($)	Std. Dev ($)
Men	1,404	6,850	260	45,000	7,355
Women	1,447	4,742	225	25,000	5,376

Data source: RAND Health and Retirement Study (HRS). Download from:
https://hrsdata.isr.umich.edu/data-products/rand-hrs-longitudinal-file-2018

Table A1: Annual contributions to defined-contribution plans by US male and female pensioners